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Electroweak hadronic contributions to $g_\mu - 2$

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Abstract: We reanalyze the two-loop electroweak hadronic contributions to the muon $g-2$ that may be enhanced by large logarithms. The present evaluation is improved over those already existing in the literature by the implementation of the current algebra Ward identities and the inclusion of the correct short-distance QCD behaviour of the relevant hadronic Green's function.

Keywords: Nonperturbative Effects, 1/N Expansion, Electromagnetic Processes and Properties, QCD.

Contents

1. Introduction

The latest result from the $g_\mu - 2$ experiment at BNL [1] reported by the E821 collaboration has triggered a renewal of interest on the theoretical prediction of the anomalous magnetic moment of the muon $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$ in the Standard Theory.[1] The attention has been focused mostly on the hadronic vacuum polarization contribution, particularly on the accuracy of its determination [3]–[7]. The major theoretical change, however, comes from a new evaluation of the dominant pion-pole contribution to the hadronic light-by-

[1]For a recent review see e.g. ref. [2] where some of the earlier references can also be found.

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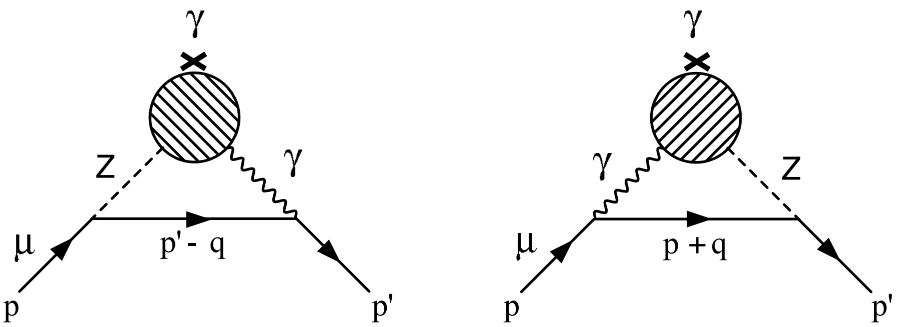

Figure 1: Feynman diagrams with the hadronic $\gamma - \gamma - Z$ vertex which contributes to the muon anomaly.

light scattering [8], which has unravelled a sign mistake in earlier calculations [9]–[13]. The correctness of the result in ref. [8] was corroborated by a renormalization group argument within the low-energy effective theory of the Standard Theory in ref. [14], with subsequent developments in refs. [15, 16] and [17]. The previous calculations have now been amended correspondingly [18, 19, 20].

Here we wish to report on a new calculation concerning an interesting class of hadronic contributions which appear at the two-loop level in the electroweak sector. They are the ones generated by the hadronic $\gamma - \gamma - Z$ vertex, with one γ and the Z-boson attached to a muon line, as illustrated by the Feynman diagrams in figure 1.

As first noticed in [21] these contributions are particularly interesting since, in principle, they can be enhanced by a large $\log(M_Z^2/m_{\rm loop}^2)$ factor, where $m_{\rm loop}$ is the relevant scale in the shaded loop in figure 1. However, in the absence of the strong interactions, there is an important cancellation between leptons and quarks within a given family, as a consequence of the anomaly-free charge assignment in the Standard Theory [22, 23]. The purpose of the present work is to analyze what is left out of this cancellation in the sector of the u, d and s quarks, where the strong interactions turn out to play a subtle role.

When dealing with the strong interaction effects, it has become a common *simplifying practice* among some theorists, to assume that the main effect of the strong interactions is *dual* to a modification of the u, d, s quark masses in the QCD lagrangian, to "constituent-like" quark masses of the order of 300 MeV for the u, d quarks and 500 MeV for the s quark. In this way the contributions from figure 1 were found to be rather small [22, 23].[2] However, in view of the expected accuracy which the new BNL experiment will eventually reach, it becomes necessary to have a more reliable determination of the size of these contributions. This has now become possible within the framework of the $1/N_c$ expansion in QCD. There are indeed recent theoretical developments of this non-perturbative analytic approach, which have been applied mostly to the calculation of hadronic weak processes,[3] but which turn out to be useful as well for the evaluation of this particular class of hadronic contributions to a_μ. This paper is dedicated to the evaluation of these contributions.

[2]Notice that ref. [22] uses a more sophisticated version of this approach where at least chiral symmetry breaking is correctly implemented [24].

[3]See ref. [25] for a recent review, and refs. [26] to [30] for details.

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The relevant terms in the lagrangian of the Standard Theory which we shall be needing are the following:

1a. **The Leptonic Neutral Current lagrangian.**

$$\mathcal{L}_{\mathrm{NC}}^{\mathrm{leptons}} = eJ_\mu^{\mathrm{em}} A^\mu - \frac{g}{2\cos\theta_W} J_\mu^{(0)} Z^\mu \,,$$

where

$$J_\mu^{\mathrm{em}} = \sum_f Q_f \bar{f}\gamma_\mu f\,, \qquad \text{and} \qquad J_\mu^{(0)} = \sum_f \bar{f}\gamma_\mu(\mathrm{v}_f - \mathrm{a}_f\gamma_5) f\,,$$

with

$$\mathrm{v}_f = T_{3f} - 2Q_f \sin^2\theta_W\,, \qquad \text{and} \qquad \mathrm{a}_f = T_{3f}\,.$$

In our case, we shall only need the muon components

$$Q_\mu = -1\,, \qquad \text{and} \qquad \mathrm{a}_\mu = -\frac{1}{2}\,;$$

i.e., the muon electromagnetic coupling and the axial coupling of the Z to the muon line:

$$-e\bar{\mu}(x)\gamma_\alpha\mu(x)A^\alpha(x) \qquad \text{and} \qquad \frac{g}{2\cos\theta_W}\frac{1}{2}\bar{\mu}(x)\gamma_\alpha\gamma_5\mu(x)Z^\alpha(x)\,.$$

1b. **The Hadronic Neutral Current lagrangian in the sector of light quarks.**

$$\mathcal{L}_{\mathrm{NC}}^{\mathrm{quarks}} = \bar{q}(x)\gamma^\mu \left[l_\mu^{(0)}\frac{1-\gamma_5}{2} + r_\mu^{(0)}\frac{1+\gamma_5}{2}\right] q(x)\,,$$

where $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ and

$$l_\mu^{(0)} = eQ_L[A_\mu - \tan\theta_W Z_\mu] + \frac{g}{2\cos\theta_W} Q_L^{(3)} Z_\mu \qquad \text{and} \qquad r_\mu^{(0)} = eQ_R[A_\mu - \tan\theta_W Z_\mu]\,.$$

In our calculation, only the hadronic electromagnetic coupling:

$$e\,\bar{q}(x)\gamma^\mu Q q(x) A_\mu\,, \qquad Q = Q_L = Q_R = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right);$$

and the hadronic axial coupling:

$$-\frac{1}{2}\frac{g}{2\cos\theta_W}\bar{q}(x)\gamma^\mu\gamma_5 Q_L^{(3)} q(x) Z_\mu(x)\,, \qquad Q_L^{(3)} = \mathrm{diag}(1, -1, -1)\,,$$

will be needed.

1c. **The coupling of the unphysical neutral Higgs field to the light quarks and to the muon.**

$$\mathcal{L}_{\Phi^0} = \frac{g}{2\cos\theta_W}\Phi^0(x) J^{\Phi^0}(x)\,, \tag{1.1}$$

with ($\mathrm{a}_u = 1/2$, $\mathrm{a}_d = \mathrm{a}_s = -1/2$)

$$J^{\Phi^0}(x) = 2\sum_{q=u,d,s} \mathrm{a}_q \frac{m_q}{M_Z}\bar{q}(x) i\gamma_5 q(x) + 2\mathrm{a}_\mu \frac{m_\mu}{M_Z}\bar{\mu}(x) i\gamma_5\mu(x)\,. \tag{1.2}$$

The hadronic electroweak vertex which we have to compute, in the notation corresponding to figure 1, is the following[4]

$$
\begin{aligned}
\langle \bar\mu(p')|V_\rho^{\rm em}(0)|\mu(p)\rangle &= \bar u(p')\Gamma_\rho(p',p)u(p) \\
&= (-ie)(-ie)\left(\frac{ig}{4\cos\theta_W}\right)\left(\frac{-ig}{4\cos\theta_W}\right)\times \\
&\quad \times \int \frac{d^4q}{(2\pi)^4}\,\frac{-i}{q^2}\,\frac{-i}{(p'-p-q)^2-M_Z^2}\bar u(p')\times \\
&\quad \times \left[\gamma^\mu \frac{i}{\not p' - \not q - m_\mu}\gamma^\nu\gamma_5 + \gamma^\nu\gamma_5\frac{i}{\not p + \not q - m_\mu}\gamma^\mu\right] u(p)\times \\
&\quad \times \int d^4x\, e^{iq.x}\int d^4y\, e^{i(p'-p-q)\cdot y}\langle\Omega|T\{V_\mu^{\rm em}(x)A_\nu^{\rm nc}(y)V_\rho^{\rm em}(0)\}|\Omega\rangle\,, \quad (1.3)
\end{aligned}
$$

where

$$
V_\mu^{\rm em}(x) = \bar q(x)\gamma_\mu\, Q\, q(x)\,, \qquad A_\nu^{\rm nc}(y) = \bar q(y)\gamma_\nu\gamma_5\, Q_L^{(3)}\, q(y)
$$

and $|\Omega\rangle$ denotes the full QCD physical vacuum. Here, for the purpose of simplicity, both the photon and Z propagators in the second line of eq. (1.3) are evaluated in the Feynman gauge. We dedicate, however, appendix A to a discussion of questions related to gauge dependence. The anomalous magnetic moment contribution from this vertex is then defined by the corresponding Pauli form factor at zero momentum transfer, i.e.,

$$
F_2(0) = \lim_{k^2\to 0} \operatorname{tr}\left\{(\not p + m_\mu)\Lambda_2^\rho(p',p)(\not p' + m_\mu)\Gamma_\rho(p',p)\right\}, \tag{1.4}
$$

where ($p' = p + k$)

$$
\Lambda_2^\rho(p',p) = \frac{m_\mu^2}{k^2}\frac{1}{4m_\mu^2-k^2}\gamma^\rho - \frac{m_\mu}{k^2}\frac{2m_\mu^2+k^2}{(4m_\mu^2-k^2)^2}(p+p')^\rho \tag{1.5}
$$

is the projector on the Pauli form factor, and m_μ denotes the muon mass.

2. The master Green's function

The contribution we want to compute is governed by the hadronic Green's function

$$
W_{\mu\nu\rho}(q,k) = \int d^4x\, e^{iq\cdot x}\int d^4y\, e^{i(k-q)\cdot y}\langle\Omega|T\{V_\mu^{\rm em}(x)A_\nu^{\rm nc}(y)V_\rho^{\rm em}(0)\}|\Omega\rangle\,, \tag{2.1}
$$

with k the incoming photon four-momentum associated with the classical external magnetic field, as illustrated in figure 2.

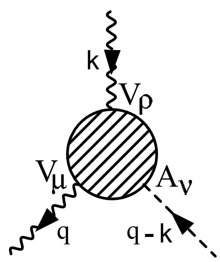

Figure 2: Symbolic representation of the $\langle VAV\rangle$ three-point function in eq. (2.1).

The general Ward identities which constrain this type of three-point functions in QCD are discussed in appendix A. In particular, $W_{\mu\nu\rho}(q,k)$ has an electroweak U(1) anomalous term. It is this term which, when lumped together with the corresponding contributions from the c quark and the e and μ leptons to the $\gamma - \gamma - Z$ loop, gives a finite gauge

[4]We use the following conventions for Dirac's γ-matrices: $\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}$, with $g_{\mu\nu}$ the Minkowski metric tensor of signature $\{+,-,-,-\}$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the totally antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ is such that $\epsilon_{0123} = +1$.

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invariant contribution to a_μ which was calculated, independently, in refs. [22, 23]. Here we are particularly interested in the contribution to a_μ from the non-anomalous part of $W_{\mu\nu\rho}(q,k)$, denoted by $\tilde{W}_{\mu\nu\rho}(q,k)$, i.e.

$$W_{\mu\nu\rho}(q,k) = W_{\mu\nu\rho}(q,k)_{\text{anomaly}} + \tilde{W}_{\mu\nu\rho}(q,k)\,, \tag{2.2}$$

where

$$W_{\mu\nu\rho}(q,k)_{\text{anomaly}} = -i\frac{N_c}{12\pi^2}\frac{4}{3}\frac{(q-k)_\nu}{(q-k)^2}\epsilon_{\mu\rho\alpha\beta}q^\alpha k^\beta\,. \tag{2.3}$$

The function $\tilde{W}_{\mu\nu\rho}(q,k)$ obeys the trivial Ward identities

$$q^\mu \tilde{W}_{\mu\nu\rho}(q,k) = 0\,, \qquad \text{and} \qquad k^\rho \tilde{W}_{\mu\nu\rho}(q,k) = 0\,, \tag{2.4}$$

but, as discussed in appendix A, it has both a longitudinal and transverse component in the axial neutral-current ν-index:

$$\tilde{W}_{\mu\nu\rho}(q,k) = \tilde{W}^{\text{long}}_{\mu\nu\rho}(q,k) + \tilde{W}^{\text{trans}}_{\mu\nu\rho}(q,k)\,.$$

Differentiating the second identity in eq. (2.4) with respect to k we get

$$\tilde{W}_{\mu\nu\rho}(q,k) = -k^\sigma\frac{\partial}{\partial k^\rho}\tilde{W}_{\mu\nu\sigma}(q,k)\,. \tag{2.5}$$

As we shall see in the next section, extracting one power of k from $\tilde{W}^{\text{trans}}_{\mu\nu\rho}(q,k)$ is all that is needed in order to obtain the corresponding contribution to the Pauli form factor at zero momentum transfer, i.e. the anomalous magnetic moment a_μ. Therefore, without loss of generality, we can set

$$\tilde{W}^{\text{trans}}_{\mu\nu\rho}(q,k) = k^\sigma W_{\mu\nu\rho\sigma}(q) + \mathcal{O}(k^2)\,,$$

where

$$W_{\mu\nu\rho\sigma}(q) = -\frac{\partial}{\partial k^\rho}\left[W_{\mu\nu\sigma}(q,k) - \frac{(q-k)_\nu}{(q-k)^2}(q-k)^{\nu'}W_{\mu\nu'\sigma}(q,k)\right]\Big|_{k\to 0}. \tag{2.6}$$

The most general pseudo-tensor $W_{\mu\nu\rho\sigma}(q)$ which satisfies the Ward identities is then constrained to have the form ($Q^2 \equiv -q^2$):[5]

$$W_{\mu\nu\rho\sigma}(q) = iW(Q^2)\,[q_\rho\epsilon_{\mu\nu\alpha\sigma}q^\alpha - q_\sigma\epsilon_{\mu\nu\alpha\rho}q^\alpha]\,. \tag{2.7}$$

As we shall see in the next section, the contribution we are looking for can be expressed as a simple integral of the function $W(Q^2)$ over the range $0 \le Q^2 \le \infty$ in the euclidean Q^2

[5] There is *a priori* another possible tensor structure which also fulfills these requirements:

$$\epsilon_{\mu\nu\rho\sigma}q^2 + q_\nu\epsilon_{\mu\rho\lambda\sigma}q^\lambda + q_\mu\epsilon_{\rho\nu\lambda\sigma}q^\lambda\,.$$

However, using the identity

$$\epsilon^{\mu\nu\rho\sigma}q^\lambda + \epsilon^{\nu\rho\sigma\lambda}q^\mu + \epsilon^{\rho\sigma\lambda\mu}q^\nu + \epsilon^{\sigma\lambda\mu\nu}q^\rho + \epsilon^{\lambda\mu\nu\rho}q^\sigma = 0\,,$$

it can easily be shown (contracting the previous identity with q_λ) that it is identical to the one in eq. (2.7).

variable. We shall show in section 4 that, in the chiral limit, this function falls as $1/Q^6$ at large Q^2 and, in the presence of massive light quarks, the large-Q^2 fall-off of $W(Q^2)$ goes as $1/Q^4$ only.

3. The contribution to the muon anomalous magnetic moment

Since what we want is $F_2(0)$ we need only to keep the projector in eq. (1.5) to its simplest form in an expansion in powers of momentum k,

$$\Lambda_2^\rho(p',p) = \frac{1}{4k^2}\left(\gamma^\rho - \frac{(p+p')^\rho}{2m_\mu}\right) + \cdots .$$

This is because the product

$$(\not p + m_\mu)\Lambda_2^\rho(p',p)(\not p' + m_\mu) = \frac{1}{4k^2}(\not p + m_\mu)\left[\gamma^\rho\ \not k - \left(k^\rho + \frac{p^\rho}{m_\mu}\ \not k\right)\right] + \mathcal{O}(k^0)\,,$$

which then appears in eq. (1.4), is proportional to one inverse power of k. On the other hand, gauge invariance forces the Green's function $W_{\mu\nu\rho}(q,k)$ to be proportional to one power of k at least. Altogether, this gives the required powers of k which are needed to define $F_2(0)$. We can, therefore, simplify significantly the calculation of eq. (1.4) to that of the simpler expression:

$$\begin{aligned} F_2(0) &= (-e^2)\frac{g^2}{16\cos^2\theta_W}\frac{1}{M_Z^2}\lim_{k^2\to 0}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\left(\frac{M_Z^2}{q^2-M_Z^2}\right)\times \\ &\times\frac{1}{4k^2}\,\mathrm{tr}\Bigg\{(\not p + m_\mu)\left[\gamma^\rho\ \not k - \left(k^\rho + \frac{p^\rho}{m_\mu}\ \not k\right)\right]\times \\ &\qquad\times\left[\gamma^\mu\frac{(\not p - \not q + m_\mu)}{q^2-2q\cdot p}\gamma^\nu\gamma_5 + \gamma^\nu\gamma_5\frac{(\not p + \not q + m_\mu)}{q^2+2q\cdot p}\gamma^\mu\right]\Bigg\}\times \\ &\times\left[-\frac{N_c}{12\pi^2}\frac{4}{3}\frac{q_\nu}{q^2}\epsilon_{\mu\rho\alpha\sigma}q^\alpha k^\sigma - i\tilde{W}^{\mathrm{long}}_{\mu\nu\rho}(q,k) + k^\sigma\left[q_\rho\epsilon_{\mu\nu\alpha\sigma}q^\alpha - q_\sigma\epsilon_{\mu\nu\alpha\rho}q^\alpha\right]W(Q^2)\right], \end{aligned} \tag{3.1}$$

where the first term in the last line of eq. (3.1) is the contribution from the anomaly to the $\langle VAV\rangle$ Green's function, already discussed in ref. [22]. The second and third terms are the non-anomalous contributions from the longitudinal and transverse components of the same Green's function. Let us first simplify, still further, the last contribution.

The integral over the four-momentum q from the last term (the transverse component), is convergent in the infrared because $W(Q^2)$ has no Goldstone pole at $Q^2\to 0$ and in the ultraviolet because of the QCD short-distance behaviour of $W(Q^2)$, which we shall later discuss. Therefore, to leading order in powers of the lepton mass, we can perform the trace and then the integration over the angles, setting

$$\frac{1}{q^2\pm 2q\cdot p}\to\frac{1}{q^2}\,, \tag{3.2}$$

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in eq. (3.1). The integral over the solid angle $d\Omega_q$ becomes trivial, and the transverse contribution is then given by the expression

$$
\begin{aligned}
F_2(0)|_{\rm trans} = & \frac{g^2}{16\cos^2\theta_W}\frac{e^2}{M_Z^2}\lim_{k^2\to 0}\frac{i}{16\pi^2}\int_0^\infty dQ^2 Q^2 \frac{1}{Q^4}\left(\frac{M_Z^2}{Q^2+M_Z^2}\right) Q^2 W(Q^2)\times \\
& \times\frac{-1}{8k^2}\,{\rm tr}\Big\{(\not p+m_\mu)\left[\gamma^\rho\not k-\left(k^\rho+\frac{p^\rho}{m_\mu}\not k\right)\right]\times \\
& \qquad\times\left[\gamma^\mu(\not p+m_\mu)\gamma^\nu\gamma_5+\gamma^\nu\gamma_5(\not p+m_\mu)\gamma^\mu\right]\Big\}k^\sigma\epsilon_{\mu\nu\rho\sigma} \\
& +\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right).
\end{aligned}
\tag{3.3}
$$

The remaining algebra leads to a remarkably simple expression:

$$
F_2(0)|_{\rm trans} = (-e^2)\frac{g^2}{16\cos^2\theta_W}\left(\frac{m_\mu^2}{M_Z^2}\right)\frac{1}{4\pi^2}\int_0^\infty dQ^2\left(\frac{M_Z^2}{Q^2+M_Z^2}\right)W(Q^2)\,. \tag{3.4}
$$

The integration of the anomalous term in eq. (3.1) is in fact slightly more complicated, because there we cannot simplify the muon propagators as in eq. (3.2). It is then useful to combine denominators using two Feynman parameters:

$$
\frac{1}{(q^2)^2(q^2-M_Z^2)(q^2\pm 2q\cdot p)} = 6\int_0^1 dx x^2\int_0^1 dy y\frac{1}{[(q-l)^2-R^2]^4}\,,
$$

where

$$
-l=\pm p(1-x) \qquad \text{and} \qquad R^2 = M_Z^2\, x(1-y)+m_\mu^2(1-x)^2\,.
$$

Then, after the shift $q\to q+l$ of the integration four-momentum, and upon neglecting terms of $\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)$ one obtains the result (in the Feynman gauge)

$$
\begin{aligned}
F_2(0)|_{\rm anom} = & \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\frac{N_c}{3}\frac{4}{3}\times \\
& \times\int_0^1 dx\,x^2\int_0^1 dy\,y\left[\frac{1+3(1-x)}{x(1-y)+\frac{m_\mu^2}{M_Z^2}(1-x)^2}-\frac{m_\mu^2}{M_Z^2}\frac{(1-x)^3}{\left[x(1-y)+\frac{m_\mu^2}{M_Z^2}(1-x)^2\right]^2}\right], \\
= & \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}N_c\left\{\frac{4}{9}\log\frac{M_Z^2}{m_\mu^2}+\frac{2}{9}+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)\right\}, \\
= & \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times 18.69\,,
\end{aligned}
\tag{3.5}
$$

in agreement with the result[6] in ref. [22]. The evaluation of $F_2(0)$ in an arbitrary gauge, with a discussion of the physics behind it, can be found in appendix A.

[6]The constant term inside the bracket in eq. (20) of ref. [22] should be 2/3 instead of 4/9.

4. Constraints on the function $W(Q^2)$ from the operator product expansion (OPE)

In order to perform the integral over Q^2 in eq. (3.4) we require information on the function $W(Q^2)$. We shall now discuss the constraints on this function, at large Q^2 values, which follow from QCD at short-distances. For pedagogical reasons, we shall first discuss the hadronic contributions in the chiral limit, where the light quark masses are neglected. However, the leading effect due to the explicit chiral symmetry breaking induced by the finite light quark masses, turns out to be important and rather subtle. It is at the origin of a conceptual discrepancy with the treatment reported in ref. [23]. We shall, therefore, discuss as well the contributions from explicit breaking, later, in section 6.

Since one of the photons is on shell, the relevant short-distance behaviour we are looking for is the one given by the OPE of the two currents $V_\mu^{\mathrm{em}}(x)$ and $A_\nu^{\mathrm{nc}}(y)$ in the three-point function $\tilde{W}_{\mu\nu\rho}(q,k)$ in eq. (2.1) when $x \to y$, because these are the currents where the hard virtual momentum q can get through [31] (from here onwards, the subscript *non-anomalous* in T-products will be understood). This three-point function is represented symbolically in figure 2. The quantity we are interested in is defined by the limit

$$\mathcal{U}_{\mu\nu}(q) = \lim_{q\to\infty} \int d^4z e^{iq.z} T\left\{V_\mu^{\mathrm{em}}(z) A_\nu^{\mathrm{nc}}(0)\right\}, \tag{4.1}$$

and the relevant term in the OPE of these two currents is the one with a tensor structure

$$\mathcal{U}_{\mu\nu}(q) = i\left[q_\delta \epsilon_{\mu\nu\alpha\beta} q^\alpha - q_\beta \epsilon_{\mu\nu\alpha\delta} q^\alpha\right] \mathcal{C} \frac{\mathcal{O}^{\beta\delta}(0)}{(Q^2)^p} + \cdots, \tag{4.2}$$

where $\mathcal{C}$ is a dimensionless constant to be calculated, p denotes the lowest possible power and the ellipsis stands for other contributions proportional to higher powers of $1/Q^2$ and/or to other tensor structures which cannot contribute to the muon anomaly. The dimension of the $\mathcal{O}^{\beta\delta}$ operator is such that:

$$d\left[\mathcal{O}^{\beta\delta}\right] - 2p = 0;$$

i.e., $\mathcal{O}^{\beta\delta}$ must be an operator of *even* dimension, and antisymmetric in $\beta\delta$.

There are in fact two types of potential contributions to the r.h.s. of eq. (4.2) which, physics-wise, are generated by the emission of *soft virtual gluons* and *hard virtual gluons* in the T-product in eq. (4.1). As discussed in appendix B, there is no possible contribution from *soft gluons*, on symmetry grounds, with the tensor structure of eq. (4.2) and with a power $p \leq 3$. The leading contribution, which appears with a power $p = 3$, comes from the perturbation theory expansion in *hard virtual gluons* and it is the one shown in the diagrams of figure 3.

The details of the calculation of the contribution from *hard virtual gluons* to $\mathcal{U}_{\mu\nu}(q)$ in eq. (4.1) are explained in appendix B where it is shown that,

$$\mathcal{U}_{\mu\nu}(q) = i\left[q_\delta \epsilon_{\mu\nu\alpha\beta} q^\alpha - q_\beta \epsilon_{\mu\nu\alpha\delta} q^\alpha\right] \left(-2\pi^2 \frac{\alpha_{\mathrm{s}}}{\pi}\right) \frac{\mathcal{O}^{\beta\delta}(0)}{(Q^2)^3} + \cdots, \tag{4.3}$$

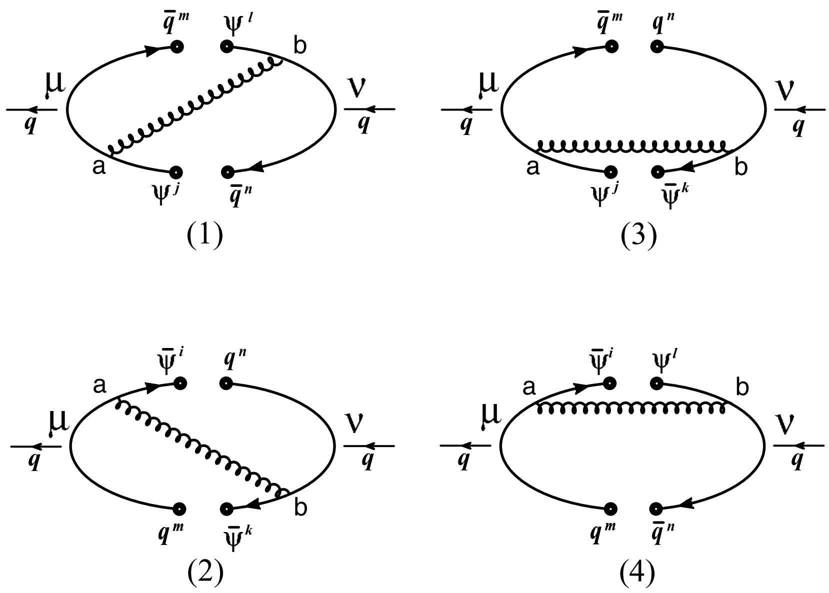

Figure 3: Feynman diagrams showing the hard gluon exchanges which contribute to the leading $1/Q^6$ behaviour of the operator $\mathcal{U}_{\mu\nu}(q)$ in eq. (4.1).

and where the operator $\mathcal{O}^{\beta\delta}(0)$ is found to be the four-quark operator:

$$\mathcal{O}^{\beta\delta}(0) = \left[\frac{2}{3}\left(\bar{u}\sigma^{\beta\delta}u\right)(\bar{u}u) + \frac{1}{3}\left(\bar{d}\sigma^{\beta\delta}d\right)\left(\bar{d}d\right) + \frac{1}{3}\left(\bar{s}\sigma^{\beta\delta}s\right)(\bar{s}s)\right](0)\,.$$

We find, therefore, that the constant $\mathcal{C}$ in eq. (4.2), to lowest order in perturbative QCD is

$$\mathcal{C} = -2\pi^2\frac{\alpha_{\rm s}}{\pi} + \mathcal{O}\left(\alpha_{\rm s}^2\right).$$

We are now in the position to evaluate the high-Q^2 behaviour of the invariant function $W(Q^2)$ in eq. (2.7). For that we first insert the result obtained in eq. (4.3) into the r.h.s. of eq. (2.6). What appears then is the two-point function

$$\Psi^{\beta\delta}{}_{\rho}(k) = \int d^4y e^{-ik\cdot y}\langle\Omega\,|T\left\{\mathcal{O}^{\beta\delta}(0)V_{\rho}^{\rm em}(y)\right\}|\Omega\rangle\,.$$

In our case, the four-momentum k in this two-point function is a *soft momentum*; and in fact, what we need for our purposes is only the term linear in this soft momentum, i.e.

$$-\frac{\partial}{\partial k^{\rho}}\Psi^{\beta\delta}{}_{\sigma}(k)\Big|_{k\to 0} = i\int d^4y\; y_{\rho}\;\langle\Omega\,|T\left\{\mathcal{O}^{\beta\delta}(0)V_{\sigma}^{\rm em}(y)\right\}|\Omega\rangle \qquad (4.4)$$

$$= i\langle\bar{\psi}\psi\rangle_0\int d^4y\; y_{\rho}\;\langle\Omega\,|T\left\{V_{\sigma}^{\rm em}(y)\;\left(\bar{q}(0)\sigma^{\beta\delta}QQ_L^{(3)}q(0)\right)\right\}|\Omega\rangle\,, \qquad (4.5)$$

where in going from eq. (4.4) to eq. (4.5) we have used the $1/N_c$-expansion, keeping only the leading contribution, and SU(3) symmetry for the single flavour vacuum condensate $\langle\bar{\psi}\psi\rangle_0$. Our problem is then reduced to the one of evaluating the behaviour of the $\langle VT\rangle$ correlation function which appears in the r.h.s. of eq. (4.5) at a small momentum transfer.

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On general grounds (conservation of the vector current, parity invariance and SU(3) symmetry) and in the chiral limit, the generic $\langle VT\rangle$ correlation function

$$\int d^4y\ e^{ik\cdot y}\langle\Omega|T\left\{\left(\bar{\psi}\gamma_\sigma\frac{\lambda^a}{2}\psi\right)(y)\ \left(\bar{\psi}\sigma^{\beta\delta}\frac{\lambda^b}{2}\psi\right)(0)\right\}|\Omega\rangle = (k^\beta\delta^\delta_\sigma - k^\delta\delta^\beta_\sigma)\ \delta^{ab}\ \Pi_{VT}(k^2)\,, \tag{4.6}$$

depends on one invariant function $\Pi_{VT}(k^2)$, where λ^a denotes the Gell-Mann matrices, normalized as $\mathrm{tr}(\lambda^a\lambda^b) = 2\delta^{ab}$, with $\lambda^0 = (\sqrt{2/3})\,\mathbf{1}$, and $a = 0, 1, \ldots, 8$. Using the relations

$$Q = \frac{\lambda^3}{2} + \frac{1}{\sqrt{3}}\frac{\lambda^8}{2}\,, \qquad \text{and} \qquad QQ_L^{(3)} = \frac{1}{3}\left(\frac{\lambda^3}{2} + \frac{1}{\sqrt{3}}\frac{\lambda^8}{2} + 4\sqrt{\frac{2}{3}}\frac{\lambda^0}{2}\right),$$

we can write our result in eq. (4.5) in terms of $\Pi_{VT}(0)$ only, as follows:

$$-\frac{\partial}{\partial k^\rho}\Psi^{\beta\delta}{}_\sigma(k)\Big|_{k\to 0} = \frac{1}{3}\left(1+\frac{1}{3}\right)\langle\bar{\psi}\psi\rangle_0(\delta^\beta_\rho\delta^\delta_\sigma - \delta^\delta_\rho\delta^\beta_\sigma)\Pi_{VT}(0)\,.$$

The function $\Pi_{VT}(k^2)$, in the *minimal hadronic approximation* (MHA) to large-N_c QCD, has been evaluated in ref. [32]. In this approximation, it has the simple pole form

$$\Pi_{VT}(k^2)|_{\mathrm{MHA}} = -\langle\bar{\psi}\psi\rangle_0\frac{1}{k^2 - M_\rho^2}\,,$$

with M_ρ the mass of the lowest vector state in the large-N_c QCD spectrum. The asymptotic behaviour of the function $W(Q^2)$ at large Q^2 values is then fixed, in the MHA, with the result

$$\lim_{Q^2\to\infty} W(Q^2)|_{\mathrm{MHA}} = \frac{16}{9}\pi^2\frac{\alpha_s}{\pi}\frac{1}{M_\rho^2}\frac{\langle\bar{\psi}\psi\rangle_0^2}{Q^6}\,. \tag{4.7}$$

5. The function $W(Q^2)$ in the MHA to large-N_c QCD and in the chiral limit

Since $W(Q^2)$ has dimensions of an inverse squared mass, it is convenient to define

$$W(Q^2) = \frac{1}{M_\rho^2}w(z)\,, \qquad \text{with } z \equiv \frac{Q^2}{M_\rho^2}\,. \tag{5.1}$$

The function $w(z)$ in large-N_c QCD is a meromorphic function in the complex z-variable. It has the pole structure shown by the Feynman diagrams in figure 4, where the ρ_i denote vector narrow states and the a_j axial-vector states.

Using partial fractions, we can write in full generality

$$w(z) = \sum_R \frac{\alpha_R}{z + z_R}\,, \qquad \text{with } z_R = \frac{M_R^2}{M_\rho^2}\,,$$

and M_R the mass of the R narrow state. The short-distance constraints on the unknown residues α_R are

$$\sum_R \alpha_R = 0\,, \qquad \sum_R \alpha_R z_R = 0\,, \qquad \text{and} \qquad \sum_R \alpha_R z_R^2 = \mathcal{R}\,,$$

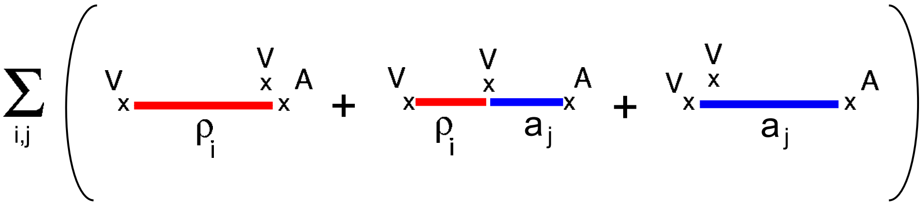

Figure 4: Feynman diagrams which contribute to the function $w(z)$ in eq. (5.1) in large-N_c QCD.

with $\mathcal{R}$ fixed by the leading term in the OPE expansion

$$\lim_{z\to\infty} w(z) = \mathcal{R}\frac{1}{z^3} + \mathcal{O}\left(\frac{1}{z^4}\right).$$

Therefore, in large-N_c QCD

$$F_2(0)|_{\rm trans} = (-e^2)\frac{g^2}{16\cos^2\theta_W}\left(\frac{m_\mu^2}{M_Z^2}\right)\frac{1}{4\pi^2}\int_0^\infty dz \frac{1}{1+\frac{M_\rho^2}{M_Z^2}z}\sum_R \frac{\alpha_R}{z+z_R}\,. \tag{5.2}$$

The minimum number p of poles required to have the same asymptotic behaviour as the OPE result in eq. (4.7) is $p = 3$. This minimum number of poles corresponds in this case to the *minimal hadronic approximation* to large-N_c QCD, which we have used already in several other calculations.[7] In this approximation, the expansion of the meromorphic function $w(z)$ in rational functions[8] is limited to the first *three poles*, and can be written as follows:

$$w(z) = \frac{\alpha_1}{z+1} + \frac{\beta_1}{z+1/g_A} + \frac{\alpha_2}{z+r'}\,, \tag{5.3}$$

with α_1, β_1 and α_2 unknown residues, which we shall determine readily, and

$$g_A = \frac{M_\rho^2}{M_{A_1}^2} \qquad \text{and} \qquad r' = \frac{M_{\rho'}^2}{M_\rho^2}\,.$$

As we have seen, the OPE evaluated in the MHA, fixes the value of $\mathcal{R}$ to

$$\mathcal{R} = \frac{16}{9}\pi^2\frac{\alpha_s}{\pi}\frac{\langle\bar\psi\psi\rangle_0^2}{M_\rho^6}\,. \tag{5.4}$$

There follow then three short-distance constraints on the three unknown residues α_1, α_2 and β_1:

$$\begin{aligned}\alpha_1 + \beta_1 + \alpha_2 &= 0\,,\\ \alpha_1 + \frac{1}{g_A}\beta_1 + r'\alpha_2 &= 0\,,\\ \alpha_1 + \frac{1}{g_A^2}\beta_1 + r'^2\alpha_2 &= \mathcal{R}\,,\end{aligned}$$

[7] See ref. [25] for a recent review, and refs. [26] to [30] for details.

[8] Recall the Mittag-Leffler theorem for meromorphic functions, see e.g. ref. [33, section 7.4].

with the solution:

$$\alpha_1 = \mathcal{R}\frac{g_A}{(1-g_A)(r'-1)}\,,$$
$$\alpha_2 = -\mathcal{R}\frac{g_A}{(1-g_A r')(r'-1)}\,,$$
$$\beta_1 = \mathcal{R}\frac{g_A^2}{(1-g_A)(1-g_A r')}\,.$$

In particular

$$w(0) = \alpha_1 + g_A\beta_1 + \frac{1}{r'}\alpha_2 = \mathcal{R}\frac{g_A}{r'}\,.$$

The function $w(z)$ is then fully determined and the integral which defines $F_2(0)$ in eq. (3.3) can be done. The shape of the function $w(z)$ normalized to its value at the origin is the continuous red curve shown in figure 5 below. As expected, it is a sharply decreasing function in z.

At this stage, it is perhaps useful to compare the function $W(Q^2)$ we find in the MHA to large-N_c QCD with the corresponding function, evaluated in the constituent chiral quark model (χQM), which was used in ref. [22] to obtain $F_2(0)|_{\rm trans}$ as an estimate of the error in $F_2(0)$. The function $W(Q^2)$ in the χQM has a simple parametric form:

$$W_{\chi\rm QM}(Q^2) = \frac{N_c}{12\pi^2}\frac{8}{3}g_A\int_0^1 dx\int_0^{1-x}dy\frac{xy-y(1-y)}{M_Q^2+Q^2y(1-y)}\,, \tag{5.5}$$

where $g_A \simeq 1/2$ is the axial-coupling of the constituent quark.[9] With the same normalization as in eq. (5.1), i.e.,

$$W_{\chi\rm QM}(Q^2) = \frac{1}{M_\rho^2}w_{\chi\rm QM}(z)\,, \qquad \text{and} \qquad z \equiv \frac{Q^2}{M_\rho^2}\,, \tag{5.6}$$

we have

$$\begin{aligned} w_{\chi\rm QM}(z) &= \frac{N_c}{12\pi^2}\frac{8}{3}g_A\frac{M_\rho^2}{M_Q^2}\times\int_0^1 dx\int_0^{1-x}dy\frac{xy-y(1-y)}{1+\frac{Q^2}{M_Q^2}y(1-y)} \\ &= \frac{-N_c}{12\pi^2}\frac{8}{3}g_A\frac{M_\rho^2}{4Q^2}\left\{1+\frac{M_Q^2}{Q^2}\frac{2}{\sqrt{1+4M_Q^2/Q^2}}\log\frac{\sqrt{1+4M_Q^2/Q^2}-1}{\sqrt{1+4M_Q^2/Q^2}+1}\right\}. \end{aligned} \tag{5.7}$$

The asymptotic behaviours of this function are:

$$\lim_{z\to 0} w_{\chi\rm QM}(z) = \frac{-N_c}{12\pi^2}g_A\frac{M_\rho^2}{9M_Q^2}+\mathcal{O}\left(\frac{Q^2}{M_Q^2}\right),$$

and

$$\lim_{z\to\infty} w_{\chi\rm QM}(z) = \frac{-N_c}{12\pi^2}\frac{2}{3}g_A\frac{M_\rho^2}{Q^2}+\mathcal{O}\left[\left(\frac{M_Q^2}{Q^2}\right)^2\log\frac{Q^2}{M_Q^2}\right].$$

[9] See the second reference in [24].

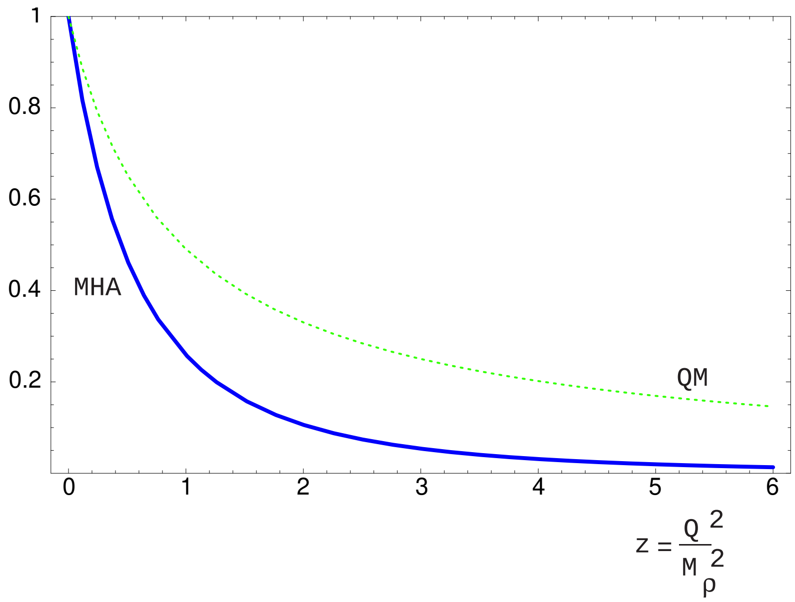

Figure 5: Shape of the functions $w(z)$ in eq. (5.1) (the continuous curve) and $w_{\chi QM}(z)$ in eq. (5.6) (the dotted curve) normalized to their respective values at the origin.

The χQM not only fails to reproduce the large Q^2 behaviour of the OPE in QCD, but it has the opposite sign to the MHA to large-N_c QCD in the chiral limit. Notice that, even though one is interested in the large-Q^2 limit, there is one photon whose momentum k^2 vanishes (see figure 2). This makes the Green's function $W(Q^2)$ in eq. (2.7) a nonperturbative object which cannot be calculated in terms of free quarks, such as those of the χQM.

The shape of the function $w_{\chi \mathrm{QM}}(z)$ normalized to its value at the origin is the dashed curve plotted in figure 5. It is yet another example of a constituent chiral quark model prediction which deviates substantially from the short-distance behaviour of QCD. The resulting contribution to $F_2(0)|_{\mathrm{trans}}$ in the χQM is as follows:

$$\begin{aligned} F_2(0)\Big|_{\mathrm{trans}}^{\chi\mathrm{QM}} &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\,\frac{\alpha}{\pi}\times\frac{N_c}{3}g_A\left[\frac{2}{3}\log\frac{M_Z^2}{M_Q^2}-\frac{4}{3}+\mathcal{O}\left(\frac{M_Q^2}{M_Z^2}\log\frac{M_Z^2}{M_Q^2}\right)\right], \\ &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times 3.08\,, \qquad \text{for } M_Q = 330\,\mathrm{MeV} \quad \text{and} \quad g_A=\frac{1}{2}\,. \end{aligned} \tag{5.8}$$

Amusingly, the integral in eq. (5.2), in the MHA to large-N_c QCD can be easily done analytically. Upon neglecting terms of $\mathcal{O}(M_{\rho'}^2/M_Z^2)$, the result reads

$$F_2(0)|_{\mathrm{trans}}^{\mathrm{MHA}} = -\frac{G_{\mathrm{F}}m_\mu^2}{\sqrt{2}}\frac{\alpha}{2\pi}\mathcal{R}\left[\frac{g_A}{(1-g_Ar')(r'-1)}\log r' + \frac{g_A^2}{(1-g_A)(1-g_Ar')}\log g_A\right].$$

Numerically, for $g_A = 1/2$ and $r' = 3.5$, we find

$$F_2(0)|_{\mathrm{trans}}^{\mathrm{MHA}} = -\frac{G_{\mathrm{F}}m_\mu^2}{\sqrt{2}}\frac{\alpha}{2\pi}\mathcal{R}\times 0.128\,. \tag{5.9}$$

In order to evaluate, in terms of hadronic parameters, the short-distance residue $\mathcal{R}$ in eq. (5.4), we observe that the same factor $\sim \frac{\alpha_s}{\pi}\langle\bar{\psi}\psi\rangle_0^2$ appears in the OPE of the $\langle LR\rangle$ correlation function where it has been shown [34] that in large-N_c QCD

$$\frac{\alpha_s}{4\pi}\langle\bar{\psi}\psi\rangle_0^2 = \frac{1}{16\pi^2}\left(\sum_j f_{A_j}^2 M_{A_j}^6 - \sum_i f_{V_i}^2 M_{V_i}^6\right).$$

In this case, the MHA corresponds to a saturation with a vector state and an axial-vector state only, with the result

$$\left.\frac{\alpha_s}{4\pi}\langle\bar{\psi}\psi\rangle_0^2\right|_{\mathrm{MHA}} = \frac{1}{16\pi^2}\left(f_{A_1}^2 M_{A_1}^6 - f_\rho^2 M_\rho^6\right). \tag{5.10}$$

Using this relation, as well as the familiar vector dominance expressions [35]:

$$f_\rho^2 = 2\frac{F_0^2}{M_\rho^2}, \qquad f_{A_1}^2 = \frac{F_0^2}{M_{A_1}^2}, \qquad \text{and} \qquad M_\rho^2 = g_A\, M_{A_1}^2,$$

which as explained in ref. [25] can also be viewed as predictions of the MHA to large-N_c QCD, our hadronic estimate for the residue $\mathcal{R}$ is then

$$\mathcal{R} = \frac{8}{9}\frac{F_0^2}{M_\rho^2},$$

with F_0 the pion coupling constant in the chiral limit ($F_0 = 87\,\mathrm{MeV}$). This estimate allows for a neat comparison with the contributions calculated in refs. [22, 23], since we now have

$$\begin{aligned} F_2(0)|_{\mathrm{trans}}^{\mathrm{MHA}} &= -\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi} \times \left(\frac{2}{9}\times 0.128\right)\frac{16\pi^2 F_0^2}{M_\rho^2} \\ &= -\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times 0.06, \end{aligned} \tag{5.11}$$

Even allowing for a substantial error in our determination of $\langle\bar{\psi}\psi\rangle_0$ in eq. (5.10), we obtain, in the chiral limit, a much smaller contribution than the one predicted by the χQM in eq. (5.8).

6. Beyond the chiral limit

As explained in appendix A, the axial Ward identity in eq. (A.3), in the presence of light quark mass terms in the QCD lagrangian, is no longer given by the anomaly alone. As a result, the non-anomalous part of the Green's function $W_{\mu\nu\rho}(q,k)$ in eq. (2.1), which we have been denoting by $\tilde{W}_{\mu\nu\rho}(q,k)$ (see eq. (2.2)) is no longer fully transverse. It acquires a longitudinal component

$$\tilde{W}_{\mu\nu\rho}^{\mathrm{long}}(q,k) = -2iM_Z\frac{(q-k)_\nu}{(q-k)^2}W_{\mu\rho}^{(\Phi^0)}(q,k) \tag{6.1}$$

where

$$W_{\mu\rho}^{(\Phi^0)}(q,k) = \int d^4x e^{iq.x} \int d^4y e^{-ik.y} \langle\Omega\,|T\{V_\mu^{\rm em}(x)V_\rho^{\rm em}(y)J^{\Phi^0}(0)\}\,|\,\Omega\rangle\,,$$
$$= \epsilon_{\mu\rho\alpha\beta}\, q^\alpha(-k^\beta)\mathcal{H}^{\Phi^0}\left(q^2,k^2,(q-k)^2\right)\,, \tag{6.2}$$

and J^{Φ^0} is the current defined in eq. (1.2), where the quark masses enter, linearly, as explicit couplings. It is the third term in eq. (A.2) that we shall now be concerned with. This term, when added together with the contribution from the unphysical Higgs coupling in eqs. (1.1) and (1.2) produces an extra contribution to the muon anomaly

$$\begin{aligned}
F_2(0)|_{\rm long} = & \frac{g^2}{16\cos^2\theta_W}\frac{e^2}{M_Z^2}\lim_{k^2\to 0}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\times \\
& \times\frac{m_\mu}{2k^2}\,{\rm tr}\Bigg\{(\not p+m_\mu)\left[\gamma^\rho\not k-\left(k^\rho+\frac{p^\rho}{m_\mu}\not k\right)\right]\times \\
& \times\left[\gamma^\mu\frac{(\not p-\not q+m_\mu)}{q^2-2q\cdot p}\gamma_5+\gamma_5\frac{(\not p+\not q+m_\mu)}{q^2+2q\cdot p}\gamma^\mu\right]\Bigg\}\times \\
& \times\left[\frac{2M_Z}{q^2}2\sum_{q=u,d,s}a_q\frac{m_q}{M_Z}\int d^4x e^{iq\cdot x}\int d^4y e^{-ik\cdot y}\langle\Omega|T\{V_\mu^{\rm em}(x)V_\rho^{\rm em}(y)\bar q(0)i\gamma_5 q(0)\}|\Omega\rangle\right],
\end{aligned} \tag{6.3}$$

which we are next going to evaluate.

At the level of accuracy that we are interested in, it is sufficient to consider the contribution from the strange quark in the sum of light quark flavours in eq. (6.3), and let $m_u = m_d \to 0$. Then, the relevant hadronic three-point function, in the notation of ref. [32] is

$$(\Pi_{VVP})_{\mu\rho}(q,-k) = \int d^4x e^{iq.x}\int d^4y e^{-ik\cdot y}\langle\Omega\,|T\{V_\mu^{\rm em}(x)V_\rho^{\rm em}(y)\bar s(0)i\gamma_5 s(0)\}\,|\,\Omega\rangle$$
$$= \epsilon_{\mu\rho\alpha\beta}q^\alpha(-k^\beta)\left(\frac{4}{9}\right)\mathcal{H}_V\left(q^2,k^2,(q-k)^2\right). \tag{6.4}$$

In the MHA to large-N_c QCD, it is found [27, 32] that:

$$\mathcal{H}_V^{\rm MHA}\left(q^2,k^2,(q-k)^2\right) = -\frac{\langle\bar\psi\psi\rangle_0}{2}\frac{q^2+k^2+(q-k)^2-\frac{N_c}{4\pi^2}\frac{M_V^4}{F_0^2}}{(q^2-M_V^2)(k^2-M_V^2)[(q-k)^2-\tilde M^2]}\,, \tag{6.5}$$

where, in the denominator, we have kept a pseudoscalar Goldstone mass $\tilde M^2$ in order to handle the infrared logarithmic divergence which, otherwise, would appear after integration over the q-momentum in (6.3). Here, we are concerned with the limit of eq. (6.5) when k is soft. Then

$$\lim_{k\to 0}\mathcal{H}_V^{\rm MHA}\left(q^2,k^2,(q-k)^2\right) = \frac{\langle\bar\psi\psi\rangle_0}{M_V^2-\tilde M^2}\left\{\frac{1-\frac{N_c}{8\pi^2}\frac{M_V^2}{F_0^2}}{q^2-M_V^2}+\frac{\frac{N_c}{8\pi^2}\frac{M_V^2}{F_0^2}-\frac{\tilde M^2}{M_V^2}}{q^2-\tilde M^2}\right\}. \tag{6.6}$$

The integral over the momentum q can now be made, very much the same way as the calculation of the anomalous contribution in eq. (3.5), with the result

$$\begin{aligned} F_2(0)|_{\rm long} = & -\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\frac{4}{9}\frac{16\pi^2 m_s\langle\bar\psi\psi\rangle_0}{M_V^4}\frac{1}{1-\frac{\tilde M^2}{M_V^2}}\int_0^1 dx x^2\int_0^1 dy y \times \\ & \times\left\{\left(1-\frac{N_c}{8\pi^2}\frac{M_V^2}{F_0^2}\right)\frac{1}{x(1-y)+\frac{m_\mu^2}{M_V^2}(1-x)^2}\left(1-\frac{m_\mu^2}{M_V^2}\frac{(1-x)^2}{x(1-y)+\frac{m_\mu^2}{M_V^2}(1-x)^2}\right)+\right. \\ & +\left(\frac{N_c}{8\pi^2}\frac{M_V^2}{F_0^2}-\frac{\tilde M^2}{M_V^2}\right)\frac{M_V^2}{\tilde M^2}\frac{1}{x(1-y)+\frac{m_\mu^2}{\tilde M^2}(1-x)^2}\times \\ & \left.\times\left(1-\frac{m_\mu^2}{\tilde M^2}\frac{(1-x)^2}{x(1-y)+\frac{m_\mu^2}{\tilde M^2}(1-x)^2}\right)\right\}. \end{aligned} \tag{6.7}$$

In the approximation where $m_\mu^2 \ll M_V^2$, $m_\mu^2 \ll \tilde M^2$, and keeping only logarithmic terms and constant terms, we get

$$\begin{aligned} F_2(0)|_{\rm long} = & \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\frac{4}{9}\frac{8\pi^2F_0^2}{M_V^2}\times \\ & \times\left\{\frac{N_c}{8\pi^2}\frac{M_V^2}{F_0^2}\frac{M_K^2}{\tilde M^2}\left(\log\frac{\tilde M^2}{m_\mu^2}+\frac{1}{2}\right)+\left[1-\frac{N_c}{8\pi^2}\frac{M_V^2}{F_0^2}\right]\frac{M_K^2}{M_V^2}\frac{M_V^2}{M_V^2-\tilde M^2}\log\frac{M_V^2}{\tilde M^2}\right\}, \end{aligned} \tag{6.8}$$

where we have used the current algebra relation $m_s\langle\bar\psi\psi\rangle_0 \simeq -F_0^2M_K^2$. For the numerical evaluation, we take $F_0 = 0.087\,\mathrm{GeV}$, $M_K = 0.498\,\mathrm{GeV}$, $M_V = 0.770\,\mathrm{GeV}$ and let $\tilde M$ vary in the range $\frac{2}{3}M_K^2 \le \tilde M^2 \le \frac{4}{3}M_K^2$, which includes the values of the η and η' masses induced by $m_s \neq 0$. This results in the value

$$F_2(0)|_{\rm long} = \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(4.57\pm1.17\pm1.37)\,, \tag{6.9}$$

where the first error is the one from the variation of $\tilde M$ and the second one is a 30% systematic error from the MHA approach. This result represents a *positive* contribution, smaller than the contribution from the anomaly, but much larger than the one from the transverse part of the hadronic $\langle VAV\rangle$ Green's function in the chiral limit.

Finally, we still have to consider possible chiral corrections in the evaluation of the transverse term $\tilde W_{\mu\nu\rho}^{\rm trans}(q,k)$; i.e., terms linear in quark masses which may contribute to the invariant function $W(Q^2)$ in eq. (3.4). The chiral corrections which are potentially important are those which modify the asymptotic behaviour of the function $W(Q^2)$ in eq. (4.7) to a less sharp behaviour. The calculation of the leading behaviour, $1/Q^4$ in this case, is technically rather involved and we have relegated it to the appendix C, where it is shown that

$$\lim_{Q^2\to\infty} W(Q^2) = -\frac{4}{9}\frac{1}{M_\rho^2}\frac{(4m_u-m_d-m_s)\langle\bar\psi\psi\rangle_0}{Q^4}+\frac{16}{9}\pi^2\frac{\alpha_{\rm s}}{\pi}\frac{1}{M_\rho^2}\frac{\langle\bar\psi\psi\rangle_0^2}{Q^6}\,, \tag{6.10}$$

where chiral corrections to the $\mathcal{O}(1/Q^6)$ term and higher order terms have been neglected.

With $W(Q^2) = \frac{1}{M_\rho^2} w(z)$ and $z \equiv \frac{Q^2}{M_\rho^2}$, we now have to construct the corresponding MHA to large-N_c QCD for the meromorphic function $w(z)$. From the leading OPE behaviour above, it follows that the minimum number of poles required is now $p = 2$. The resulting $w(z)$ function is then

$$w(z)|_{\mathrm{MHA}} = \frac{\mathcal{R}_m}{(z+1)(z+1/g_A)}, \qquad \text{with } \mathcal{R}_m = -\frac{4}{9} \frac{(4m_u - m_d - m_s)\langle\bar{\psi}\psi\rangle_0}{M_\rho^4}, \tag{6.11}$$

which leads to an anomalous magnetic moment contribution

$$F_2(0)|_{\mathrm{trans}}^{\mathrm{MHA}} = -\frac{G_{\mathrm{F}} m_\mu^2}{\sqrt{2}} \frac{\alpha}{2\pi} \frac{g_A}{1-g_A} \log\left(\frac{1}{g_A}\right) \mathcal{R}_m .$$

Neglecting m_u and m_d with respect to m_s, and using again the current algebra relation $-m_s\langle\bar{\psi}\psi\rangle_0 \simeq F_0^2 M_K^2$, we obtain (with $g_A = 1/2$, $F_0 = 0.087\,\mathrm{GeV}$, $M_K = 0.498\,\mathrm{GeV}$)

$$F_2(0)|_{\mathrm{trans}}^{\mathrm{MHA}} \simeq \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \log 2 \frac{16\pi^2 F_0^2}{9M_\rho^2} \frac{M_K^2}{M_\rho^2} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \times 0.065 . \tag{6.12}$$

In this case, however, we know that the MHA in eq. (6.11) fails to reproduce the $\mathcal{O}(1/Q^6)$ term in the OPE, which we know is there, even in the chiral limit. In order to take into account this information, we can go now one step further beyond the MHA and incorporate three poles, like in eq. (5.3) with the residues constrained now by short-distance behaviour as follows:

$$\begin{aligned} \alpha_1 + \beta_1 + \alpha_2 &= 0 , \\ \alpha_1 + \frac{1}{g_A}\beta_1 + r'\alpha_2 &= -\mathcal{R}_m , \\ \alpha_1 + \frac{1}{g_A^2}\beta_1 + r'^2\alpha_2 &= \mathcal{R} . \end{aligned}$$

The resulting $w(z)$ function in this improved case is shown in figure 6, in 10^{-3} units (the continuous curve). For the sake of comparison, we also show in the same figure and in the same units the curve corresponding to the MHA in eq. (6.11) (the dashed curve). The integral over z in the improved case can still be done analytically and, neglecting terms of $\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2}\right)$, we get

$$\begin{aligned} F_2(0)|_{\mathrm{trans}}^{\mathrm{HA3-poles}} &= -\frac{G_{\mathrm{F}} m_\mu^2}{\sqrt{2}} \frac{\alpha}{2\pi} \left\{ \frac{[\mathcal{R}_m + (\mathcal{R}+\mathcal{R}_m)g_A]\log r'}{(1-g_A r')(r'-1)} + \frac{[\mathcal{R} + (1+r')\mathcal{R}_m]\, g_A^2 \log g_A}{(1-g_A)(1-g_A r')} \right\} \\ &= \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \times (0.04 \pm 0.02) . \end{aligned} \tag{6.13}$$

As expected, the effect of including the $1/Q^6$ terms in the asymptotic behaviour of the $W(Q^2)$ function reduces the integral, but the sign is still the same as in the MHA approximation result in eq. (6.12). The final error we quote is a very generous estimate of the systematic errors.

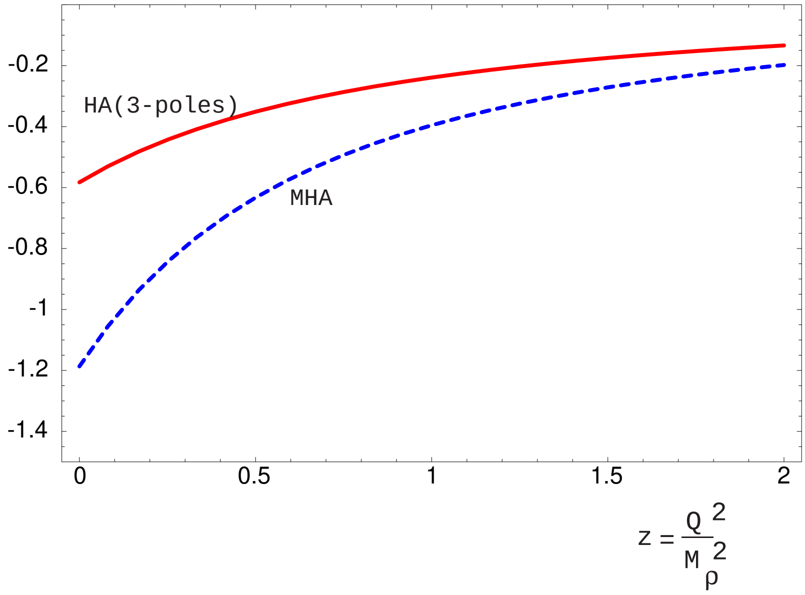

Figure 6: Shape of the functions $w(z)$ (in 10^{-3} units) in the hadronic approximation to large-N_c with 3-poles (the continuous curve) and in the minimal hadronic approximation with 2-poles in eq. (6.11) (the dashed curve).

7. Results and conclusions

The normalization reference of the electroweak contributions to the muon anomaly is the one-loop result [36]–[40]

$$a_\mu^{\mathrm{W}(1)} = \frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right],$$

where the weak mixing angle is defined by $\sin^2\theta_W = 1 - M_W^2/M_Z^2$. Numerically, with $G_{\mathrm{F}} = 1.16639(1)\times 10^{-5}\,\mathrm{GeV}^{-2}$ and $\sin^2\theta_W = 0.224$,

$$a_\mu^{\mathrm{W}(1)} = 19.48\times 10^{-10}\,,$$

while we recall that the present *world average* experimental error in the determination of the muon anomaly is

$$\Delta a_\mu|_{\mathrm{Exp.}} = \pm 15.1\times 10^{-10}\,,$$

and, hoping for a continuation of the BNL experiment, it is expected to be further reduced by still a factor of four. A theoretical effort on the evaluation of the two-loop electroweak corrections is therefore justified.

It is convenient to separate the two-loop electroweak contributions into two sets of Feynman graphs: those which contain closed fermion loops, which we denote by $a_\mu^{EW(2)}(\mathrm{ferm})$, and the others which we denote by $a_\mu^{EW(2)}(\mathrm{bos})$. In this notation, the electroweak contribution to the muon anomalous magnetic moment is

$$a_\mu^{EW} = a_\mu^{W(1)} + a_\mu^{EW(2)}(\mathrm{bos}) + a_\mu^{EW(2)}(\mathrm{ferm})\,. \tag{7.1}$$

We shall review the calculation of these two-loop contributions separately.

7.1 Bosonic contributions

The leading logarithmic terms of the two-loop electroweak bosonic corrections have been extracted using asymptotic expansion techniques (see e.g. ref. [41].) In fact, these contributions have now been evaluated analytically, in a systematic expansion in powers of $\sin^2\theta_W$, up to $\mathcal{O}[(\sin^2\theta_W)^3]$, where $\log\frac{M_W^2}{m_\mu^2}$ terms, $\log\frac{M_H^2}{M_W^2}$ terms, $\frac{M_W^2}{M_H^2}\log\frac{M_H^2}{M_W^2}$ terms, $\frac{M_W^2}{M_H^2}$ terms and constant terms are kept [42]. Using $\sin^2\theta_W = 0.224$ and $M_H = 250\,\mathrm{GeV}$, the authors of ref. [42] find

$$a_\mu^{EW(2)}(\mathrm{bos}) = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left[-5.96\log\frac{M_W^2}{m_\mu^2}+0.19\right] = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-78.9)\,. \tag{7.2}$$

7.2 Fermionic contributions

The discussion of the two-loop electroweak fermionic corrections is more delicate. As already mentioned in the introduction, because of the cancellation between lepton loops and quark loops in the electroweak U(1) anomaly, one cannot separate hadronic effects from leptonic effects any longer. In fact, as discussed in refs. [22, 23], it is this cancellation which eliminates some of the large logarithms which, were incorrectly kept in ref. [21]. It is therefore appropriate to separate the two-loop electroweak fermionic corrections into two classes: One is the class arising from Feynman diagrams like in figure 1, with both leptons and quarks in the shaded blob, including the graphs where the Z lines are replaced by Φ^0 lines, if the calculation is done in the ξ_Z-gauge. We denote this class by $a_\mu^{EW(2)}(l,q)$. The other class is defined by the rest of the diagrams, where quark loops and lepton loops can be treated separately, which we call it $a_\mu^{EW(2)}(\text{ferm-rest})$ i.e.,

$$a_\mu^{EW(2)}(\mathrm{fer}) = a_\mu^{EW(2)}(l,q) + a_\mu^{EW(2)}(\text{ferm-rest})\,.$$

The contribution from $a_\mu^{EW(2)}(\text{ferm-rest})$ brings in m_t^2/M_W^2 factors. It has been estimated, to a very good approximation, in ref. [23] with the result

$$a_\mu^{EW(2)}(\text{ferm-rest}) = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left[\frac{1}{2\sin^2\theta_W}\left(-\frac{5}{8}\frac{m_t^2}{M_W^2}-\log\frac{m_t^2}{M_W^2}-\frac{7}{3}\right)+\Delta_\mathrm{Higgs}\right],$$

where Δ_Higgs denotes the contribution from diagrams with Higgs lines, which the authors of ref. [23] estimate to be

$$\Delta_\mathrm{Higgs} = -5.5\pm 3.7\,,$$

and therefore,

$$a_\mu^{EW(2)}(\text{ferm-rest}) = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-21\,\pm\,4)\,. \tag{7.3}$$

Let us finally discuss the contributions to $a_\mu^{EW(2)}(l,q)$ which are the ones relevant to the topic of this paper. Here, it is convenient to treat the contributions from the three generations separately. The contribution from the third generation can be calculated in a straightforward way, with the result [22, 23]

$$a_\mu^{EW(2)}(\tau,t,b) = \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left[-3\log\frac{M_Z^2}{m_\tau^2}-\log\frac{M_Z^2}{m_b^2}-\frac{8}{3}\log\frac{m_t^2}{M_Z^2}+\frac{8}{3}+\mathcal{O}\left(\frac{M_Z^2}{m_t^2}\log\frac{m_t^2}{M_Z^2}\right)\right]$$
$$= \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-30.6)\,. \tag{7.4}$$

In fact the terms of $\mathcal{O}(\frac{M_Z^2}{m_t^2}\log\frac{m_t^2}{M_Z^2})$ and $\mathcal{O}(M_Z^2/m_t^2)$ have also been calculated in ref. [23]. There are in principle QCD perturbative corrections to this estimate, which have not been calculated, but the result in eq. (7.4) is good enough for the accuracy required at present.

As emphasized in ref. [22], an appropriate QCD calculation when the quark in the loop of figure 1 is a *light quark* should take into account the dominant effects of spontaneous chiral symmetry breaking. Since this involves the u and d quarks, as well as the second generation s quark, we lump together the contributions from the first and second generation with the result,

$$a_\mu^{EW(2)}(e,\mu,u,d,s,c) = \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left\{-3\log\frac{M_Z^2}{m_\mu^2}-\frac{5}{2}-3\log\frac{M_Z^2}{m_\mu^2}+\right.$$
$$+4\log\frac{M_Z^2}{m_c^2}-\frac{11}{6}+\frac{8}{9}\pi^2-8+$$
$$+\left[\frac{4}{3}\log\frac{M_Z^2}{m_\mu^2}+\frac{2}{3}+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)\right]+$$
$$\left.+4.57\pm 1.80+0.04\pm 0.02\right\} \tag{7.5}$$

$$= \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-28.5\pm 1.8)\,, \tag{7.6}$$

where the first line in eq. (7.5) shows the result from the e-loop and the second line the result from the μ-loop and the c-quark, which is treated as a heavy quark ($m_c = 1.5\,\mathrm{GeV}$). The first term in brackets in the third line is the one induced by the anomalous term in the hadronic $\langle\, VAV\rangle$ Green's function. The second term is the one induced by the leading effects of explicit chiral symmetry breaking in the non-anomalous longitudinal component of the $\langle\, VAV\rangle$ Green's function discussed in section 6 (the result in eq. (6.9)). The third term is the one induced by the transverse component of the $\langle\, VAV\rangle$ Green's function, evaluated in the presence of the light quark masses and in the HA with 3 poles to large-N_c QCD (the result in eq. (6.13)).

We want to stress that our result in eq. (7.5) for the contribution from the first and second generations of quarks and leptons is conceptually very different to the corresponding one proposed in ref. [23],

$$a_\mu^{EW(2)}(e,\mu,u,d,s,c) = \frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\left[-3\log\frac{M_Z^2}{m_\mu^2}+4\log\frac{M_Z^2}{m_u^2}-\log\frac{M_Z^2}{m_d^2}-\frac{5}{2}-6-\right.$$
$$\left.-3\log\frac{M_Z^2}{m_\mu^2}+4\log\frac{M_Z^2}{m_c^2}-\log\frac{M_Z^2}{m_s^2}-\frac{11}{6}+\frac{8}{9}\pi^2-6\right],$$

$$= \frac{G_{\rm F}}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \times (-31.9)\,. \tag{7.7}$$

where the light quarks are, *arbitrarily*, treated the same way as heavy quarks, with $m_u = m_d = 0.3\,\mathrm{GeV}$, and $m_s = 0.5\,\mathrm{GeV}$. Although, numerically, the two results turn out to be not too different, the result in eq. (7.7) follows from a hadronic model which is in contradiction with basic properties of QCD. The constituent quark model used to derive this result violates the current algebra Ward identity (A.1) derived in appendix A. Furthermore, as discussed in the text, the model does not reproduce the QCD short-distance behaviour for the underlying $\langle VAV\rangle$ Green's function. These facts are at the origin of the spurious cancellation of the $\log M_Z$ terms in eq. (7.7). A more detailed discussion of this issue can be found in appendix D.

Putting together the numerical results in eqs. (7.2), (7.3), (7.4) with the new result in eq. (7.6), we finally obtain the value

$$a_\mu^{EW} = \frac{G_{\rm F}}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_W\right)^2 - \left(\frac{\alpha}{\pi}\right)(159 \pm 4)\right] = (15.2 \pm 0.1) \times 10^{-10}\,,$$

which shows that the two-loop correction represents indeed a reduction of the one-loop result by an amount of 22%. The final error here does not include higher order electroweak estimates [43].

Acknowledgments

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A. Ward identities and gauge invariance

The hadronic Green's function in eq. (2.1) is a combination of QCD three-point functions involving the vector and axial vector currents

$$V_\mu^a(x) = \left(\overline{\psi}\,\frac{\lambda^a}{2}\,\gamma_\mu\psi\right)(x)\,, \qquad \text{and} \qquad A_\mu^a(x) = \left(\overline{\psi}\,\frac{\lambda^a}{2}\,\gamma_\mu\gamma_5\psi\right)(x)\,,$$

of the algebra of currents, where λ^a denotes the flavour Gell-Mann matrices defined after eq. (4.6). More precisely, with

$$\mathcal{T}^a_{\mu\nu;\alpha}(q_1,q_2) = i\int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)}\langle\Omega\,|\,\mathrm{T}\{V_\mu^{\rm em}(x_1)V_\nu^{\rm em}(x_2)A_\alpha^a(0)\}\,|\,\Omega\rangle\,,$$

where

$$V_\mu^{\rm em}(x) = V_\mu^3(x) + \frac{1}{\sqrt{3}}\,V_\mu^8(x)\,,$$

the relation is the following

$$\frac{i}{2}\,W_{\mu\alpha\nu}(q,k) = \mathcal{T}^3_{\mu\nu;\alpha}(q,-k) + \frac{1}{\sqrt{3}}\,\mathcal{T}^8_{\mu\nu;\alpha}(q,-k) - \frac{1}{\sqrt{6}}\,\mathcal{T}^0_{\mu\nu;\alpha}(q,-k)\,.$$

A.1 Current algebra Ward identities

For $a = 0, 3, 8$, one has the following Ward identities:[10]

$$
\begin{aligned}
(q_1 + q_2)^\alpha\, \mathcal{T}^a_{\mu\nu;\alpha}(q_1, q_2) \;=\; & -\frac{N_c}{12\pi^2}\, \mathcal{C}^a\, \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma \\
& + \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \langle \Omega \,|\, \mathrm{T}\{V^{\mathrm{em}}_\mu(x_1) V^{\mathrm{em}}_\nu(x_2) D^a_5(0)\} \,|\, \Omega\rangle \;+ \\
& + \frac{1}{8\pi^2}\, \mathrm{tr}\left(\frac{\lambda^a}{2}\right) \times \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \times \\
& \qquad \times \langle \Omega | \mathrm{T}\{V^{\mathrm{em}}_\mu(x_1) V^{\mathrm{em}}_\nu(x_2) (G\widetilde{G})(0)\} | \Omega\rangle\,,
\end{aligned} \tag{A.1}
$$

with $\mathcal{C}^3 = 1$, $\mathcal{C}^8 = 1/\sqrt{3}$, $\mathcal{C}^0 = 2\sqrt{2/3}$. Furthermore,

$$
D^a_5 \;\equiv\; d^{abc} \mathcal{M}^b P^c\,, \qquad \text{with } P^a = \overline{\psi} i \gamma_5 \frac{\lambda^a}{2}\, \psi\,,
$$

and

$$
\mathcal{M}^3 = m_u - m_d\,, \qquad \mathcal{M}^8 = \frac{1}{\sqrt{3}}(m_u + m_d - 2m_s)\,, \qquad \mathcal{M}^0 = \sqrt{\frac{2}{3}}(m_u + m_d + m_s)\,.
$$

Also

$$
(G\widetilde{G}) = \alpha_s\, \epsilon_{\mu\nu\rho\sigma} \sum_{A=1}^{8} G^{(A)\mu\nu} G^{(A)\rho\sigma}\,,
$$

with $G^{(A)\mu\nu}$ the gluon field strength tensor. Owing to the fact that

$$
\begin{aligned}
\{q_1^\mu ; q_2^\nu\} \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \langle \Omega \,|\, \mathrm{T}\{V^{\mathrm{em}}_\mu(x_1) V^{\mathrm{em}}_\nu(x_2) (G\widetilde{G})(0)\} \,|\, \Omega\rangle \;&=\; 0\,, \\
\{q_1^\mu ; q_2^\nu\} \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \langle \Omega \,|\, \mathrm{T}\{V^{\mathrm{em}}_\mu(x_1) V^{\mathrm{em}}_\nu(x_2) D^a_5(0)\} \,|\, \Omega\rangle \;&=\; 0\,,
\end{aligned}
$$

the above Ward identity can be solved as follows:

$$
\begin{aligned}
\mathcal{T}^a_{\mu\nu;\alpha}(q_1, q_2) = \frac{(q_1{+}q_2)_\alpha}{(q_1{+}q_2)^2} \Bigg\{ & -\frac{N_c}{12\pi^2}\, \mathcal{C}^a\, \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma \;+ \\
& + \frac{1}{8\pi^2}\, \mathrm{tr}\left(\frac{\lambda^a}{2}\right) \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \times \\
& \qquad \times \langle \Omega \,|\, \mathrm{T}\{V^{\mathrm{em}}_\mu(x_1) V^{\mathrm{em}}_\nu(x_2) (G\widetilde{G})(0)\} \,|\, \Omega\rangle \;+ \\
& + \int d^4x_1 d^4x_2 e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \langle \Omega | \mathrm{T}\{V^{\mathrm{em}}_\mu(x_1) V^{\mathrm{em}}_\nu(x_2) D^a_5(0)\} | \Omega\rangle \Bigg\} \;+ \\
+ \overline{\mathcal{T}}^a_{\mu\nu;\alpha}(q_1, q_2)\,, &
\end{aligned} \tag{A.2}
$$

where the tensor $\overline{\mathcal{T}}^a{}_{\mu\nu;\alpha}(q_1, q_2)$ is now completely transverse,

$$
\{q_1^\mu ; q_2^\nu ; (q_1 + q_2)^\alpha\}\, \overline{\mathcal{T}}^a_{\mu\nu;\alpha}(q_1, q_2) = 0\,.
$$

[10] Recall that we are using the following conventions: $\epsilon_{0123} = +1$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, so that $\mathrm{tr}(\gamma_5\gamma_0\gamma_1\gamma_2\gamma_3) = 4i$.

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The first term in eq. (A.2) generates the anomaly term discussed in the text. The second term is the one induced by the axial flavour-singlet U(1) anomaly. This term generates subleading contributions in the $1/N_c$-expansion with respect to the other terms; however, because of its flavour singlet nature, the *numerical size* of its contribution may not necessarily obey the *expected* $1/N_c$-power suppression. However, in our case, the source of the hadronic axial current is the one induced by the Z-boson coupling in the Standard Electroweak Theory, which has a vanishing flavour trace for each generation. Therefore, once the total sum of light and heavy quarks is taken into account, this term does not contribute to the muon anomaly.

The third term in eq. (A.2) is the one induced by the explicit breaking of chiral symmetry by the light quark masses in the QCD lagrangian. We keep it, because it will be pertinent in the forthcoming discussions of electroweak gauge invariance. Notice, for further reference, that

$$D_5^3 + \frac{1}{\sqrt{3}}\, D_5^8 - \frac{1}{\sqrt{6}}\, D_5^0 = m_u \bar{u} i\gamma_5 u - m_d \bar{d} i\gamma_5 d - m_s \bar{s} i\gamma_5 s\,.$$

A.2 Electroweak gauge invariance

Due to the conservation of the electromagnetic current, the gauge dependent terms in the photon propagator in the diagrams of figure 1 trigger naive Ward identities on the hadronic $\gamma - \gamma - Z$ vertex, and therefore we can take the Feynman gauge for the photon propagator without loss of generality, as we have done in the calculation reported in the text. By contrast, the gauge dependence of the Z-propagator requires some discussion. In the Electroweak Theory, it is convenient to rewrite the Ward identity discussed in the previous subsection in terms of the coupling of the unphysical Higgs field $\Phi^0(x)$. More precisely, in the notation used in the text,

$$(q-k)^\nu W_{\mu\nu\rho}(q,k) = -i\frac{N_c}{\pi^2}\mathcal{C}_{\text{had}}\,\epsilon_{\mu\rho\alpha\beta}\, q^\alpha k^\beta - 2iM_Z W_{\mu\rho}^{(\Phi^0)}(q,k)\,. \tag{A.3}$$

where

$$\mathcal{C}_{\text{had}} = \sum_q \mathrm{a}_q e_q^2\,, \qquad \text{with} \qquad \left.\begin{array}{llll} \mathrm{a}_q = +\frac{1}{2}\,, & \mathrm{e}_q = +\frac{2}{3}\,, & \text{for} & q = u,c,t \\ \mathrm{a}_q = -\frac{1}{2}\,, & \mathrm{e}_q = -\frac{1}{3}\,, & \text{for} & q = d,s,b \end{array}\right\}\,, \tag{A.4}$$

and

$$W_{\mu\rho}^{(\Phi^0)}(q,k) = \int d^4x e^{iq.x} \int d^4y e^{-ik.y} \langle\Omega\,|T\{V_\mu^{\text{em}}(x)V_\rho^{\text{em}}(y)J^{\Phi^0}(0)\}\,|\,\Omega\rangle\,,$$

with

$$J^{\Phi^0} = 2\sum_q \mathrm{a}_q \frac{m_q}{M_Z}\bar{q} i\gamma_5 q\,.$$

This means that in full generality, in the Electroweak Theory,

$$W_{\mu\nu\rho}(q,k) = -i\mathcal{A}\left(q^2,k^2,(q-k)^2\right)\frac{(q-k)_\nu}{(q-k)^2}\,\epsilon_{\mu\rho\alpha\beta}\, q^\alpha k^\beta + \tilde{W}_{\mu\nu\rho}^{\text{trans}}(q,k) \tag{A.5}$$

with

$$\mathcal{A}\left(q^2,k^2,(q-k)^2\right)=\frac{N_c}{\pi^2}\mathcal{C}_{\rm had}-2M_Z\ \mathcal{H}^{\Phi^0}\left(q^2,k^2,(q-k)^2\right), \tag{A.6}$$

where we have set

$$W_{\mu\rho}^{(\Phi^0)}(q,k)=\epsilon_{\mu\rho\alpha\beta}\,q^\alpha(-k^\beta)\mathcal{H}^{\Phi^0}\left(q^2,k^2,(q-k)^2\right).$$

Now, in the calculation corresponding to figure 1, the most general form of the Z-propagator, in a renormalizable linear gauge of the 't Hooft class, is

$$\mathcal{P}^{\nu\nu'}_{(\xi_Z)}(q-k)=\frac{-i}{(q-k)^2-M_Z^2}\left[g^{\nu\nu'}-(1-\xi_Z)\frac{(q-k)^\nu(q-k)^{\nu'}}{(q-k)^2-\xi_Z\,M_Z^2}\right]= \tag{A.7}$$

$$=-i\frac{\left[g^{\nu\nu'}(q-k)^2-(q-k)^\nu(q-k)^{\nu'}\right]-\xi_Z M_Z^2\left(g^{\nu\nu'}-\frac{(q-k)^\nu(q-k)^{\nu'}}{M_Z^2}\right)}{\left[(q-k)^2-M_Z^2\right]\left[(q-k)^2-\xi_Z M_Z^2\right]}. \tag{A.8}$$

The unitary gauge, which was used e.g. in ref. [22], is recovered in the limit $\xi_Z\to\infty$, which has to be taken from the start. In the text we have done the calculation in the Feynman gauge $\xi_Z=1$. Clearly, there is no contribution to the muon anomaly from the contraction of the gauge dependent piece in the Z-propagator in (A.7) with the fully transverse $\tilde{W}_{\mu\nu\rho}(q,k)$ hadronic tensor. The contribution from the non-transverse term in eq. (A.5) however, is gauge dependent. It selects the second term in the Z-propagator in (A.8), via the contraction

$$\mathcal{P}^{\nu\nu'}_{(\xi_Z)}(q-k)\times\frac{(q-k)_\nu}{(q-k)^2}=\frac{-i\xi_Z}{(q-k)^2-\xi_Z M_Z^2}\times\frac{(q-k)^{\nu'}}{(q-k)^2}.$$

As shown in eq. (A.6) the non-transverse term in eq. (A.5) has two pieces. Let us first discuss the one coming from the anomaly term. For a specific fermion f, the piece coming from the anomaly results in the expression (in the ξ_Z gauge)

$$F_2(0)\Big|^{(f)}_{\rm anom}=\frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\left(4e_f^2\mathrm{a}_f\right)\times\left[\ln\left(\frac{\xi_Z M_Z^2}{m_\mu^2}\right)+\frac{1}{2}+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)\right],$$

while in the unitarity gauge, the corresponding result is divergent and requires regularization. In dimensional regularization, with $\lambda_{\overline{\rm MS}}\equiv\frac{\mu^{d-4}}{16\pi^2}\left[\frac{1}{d-4}-\frac{1}{2}(\log 4\pi+2+\Gamma'(1))\right]$, one finds

$$F_2(0)\Big|^{(f)}_{\rm anom}=\frac{G_{\rm F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\left(4e_f^2\mathrm{a}_f\right)\times\left[\log\left(\frac{\mu^2}{M_Z^2}\right)-32\pi^2\lambda_{\overline{\rm MS}}+\log\left(\frac{M_Z^2}{m_\mu^2}\right)+\frac{1}{2}\right].$$

Recall that in these expressions, when the fermion f is a quark q, a_q and e_q have been defined in eq. (A.4). When f is a lepton l, $\mathrm{a}_l=-1/2$ and $\mathrm{e}_l=-1$. In particular, for $f=u,d,s$,

$$\sum_{u,d,s}4e_f^2\mathrm{a}_f=4N_c\frac{1}{2}\left[\left(\frac{2}{3}\right)^2+2\left(\frac{-1}{3}\right)^2(-1)\right]=\frac{N_c}{3}\frac{4}{3},$$

and in the Feynman gauge where $\xi_Z = 1$, we recover the result of eq. (3.5). Notice also that, in the Standard Theory, the ξ_Z-gauge dependence, or the $\log(\mu^2/M_Z^2) - 32\pi^2\lambda_{\overline{\rm MS}}$ dependence in the unitary gauge, cancel when quarks and leptons are taken together generation by generation.

Next, we discuss the gauge dependent piece coming from the term proportional to $\mathcal{H}^{\Phi^0}(q^2, k^2, (q-k)^2)$ in eq. (A.6), i.e. the term

$$\mathcal{P}^{\nu\nu'}_{(\xi_Z)}(q-k) \times \mathcal{H}^{\Phi^0} \frac{(q-k)_{\nu'}}{(q-k)^2} = \mathcal{H}^{\Phi^0} \frac{-i\xi_Z}{(q-k)^2 - \xi_Z M_Z^2} \frac{(q-k)^\nu}{(q-k)^2} \,.$$

This term, when acting on the leptonic line (see eq. (1.3)) triggers a trivial Ward identity:

$$\begin{aligned}(q-k)_\nu \left\{ \bar{u}(p') \left[\gamma^\mu \frac{i}{\not p' - \not q - m_\mu} \gamma^\nu \gamma_5 + \gamma^\nu \gamma_5 \frac{i}{\not p + \not q - m_\mu} \gamma^\mu \right] u(p) \right\} &= \\ = -2m_\mu \bar{u}(p') \left[\gamma^\mu \frac{i}{\not p' - \not q - m_\mu} \gamma_5 + \gamma_5 \frac{i}{\not p + \not q - m_\mu} \gamma^\mu \right] u(p) \,. & \qquad \text{(A.9)}\end{aligned}$$

The resulting expression:

$$\mathcal{H}^{\Phi^0} \frac{-i\xi_Z}{(q-k)^2 - \xi_Z M_Z^2} \frac{1}{(q-k)^2} \times 2m_\mu \bar{u}(p') \left[\gamma^\mu \frac{i}{\not p' - \not q - m_\mu} \gamma_5 + \gamma_5 \frac{i}{\not p + \not q - m_\mu} \gamma^\mu \right] u(p) \,,$$

when added together with the gauge dependent term from the unphysical Higgs contribution to the muon anomaly,

$$\mathcal{H}^{\Phi^0} \frac{i}{M_Z^2[(q-k)^2 - \xi_Z M_Z^2]} \times 2m_\mu \bar{u}(p') \left[\gamma^\mu \frac{i}{\not p' - \not q - m_\mu} \gamma_5 + \gamma_5 \frac{i}{\not p + \not q - m_\mu} \gamma^\mu \right] u(p) \,,$$

results in an overall gauge independent contribution, proportional to

$$\mathcal{H}^{\Phi^0} \frac{i}{M_Z^2 (q-k)^2} \times 2m_\mu \bar{u}(p') \left[\gamma^\mu \frac{i}{\not p' - \not q - m_\mu} \gamma_5 + \gamma_5 \frac{i}{\not p + \not q - m_\mu} \gamma^\mu \right] u(p) \,. \qquad \text{(A.10)}$$

In the unitary gauge, there are no unphysical Higgs couplings and the piece coming from the term proportional to $\mathcal{H}^{\Phi^0}$ in eq. (A.6) generates the factor

$$\mathcal{H}^{\Phi^0} \left[-i \frac{g^{\nu\nu'} - \frac{(q-k)^\nu (q-k)^{\nu'}}{M_Z^2}}{(q-k)^2 - M_Z^2} \right] \times \frac{(q-k)_{\nu'}}{(q-k)^2} = \mathcal{H}^{\Phi^0} \frac{i}{M_Z^2 (q-k)^2} (q-k)^\nu \,,$$

which, after contraction with the leptonic line, produces an expression identical to the one in eq. (A.10).

B. Technical remarks on the OPE in the chiral limit

As already mentioned in the text, the relevant OPE is the one of the product of the two currents $V_\mu^{\rm em}(x) A_\nu^{\rm nc}(y)$ in eq. (2.1) when $x \to y$, because these are the currents where the hard virtual momentum q can get through. There are two types of contributions to discuss in the chiral limit.

B.1 Contribution from soft gluons

These are the contributions generated by the term

$$\lim_{q\to\infty}\int d^4z e^{iq.z}\ T\left\{V_\mu^{\rm em}(z)A_\nu^{\rm nc}(0)\right\} \doteq\ :\bar{q}^i(0)\gamma_\mu\, Q\ \ iS^{ij}(q)\gamma_\nu\gamma_5 Q_L^{(3)}\, q^j(0):+ \\ +:\bar{q}^j(0)\gamma_\nu\gamma_5 Q_L^{(3)}\, i\tilde{S}^{ji}(q)\gamma_\mu Q\, q^i(0): \qquad \text{(B.1)}$$

in the perturbative Wick-expansion (i,j denote quark color indices) when the short-distance expansion of the quark propagator[11] is inserted:

$$iS^{ij}(q) = \int d^4z\, e^{iq\cdot z}\, iS^{ij}(z,0) \qquad \text{(B.2)}$$

$$= \delta^{ij}\frac{i}{\not q} - i\frac{q^\alpha}{q^4}g_s\tilde{G}^{ij}_{\alpha\beta}\gamma^\beta\gamma_5 + \\ +\frac{2i}{3}g_s\frac{1}{q^6}\Big[q^2D^\alpha G^{ij}_{\alpha\beta}\gamma^\beta - \not q D^\alpha G^{ij}_{\alpha\beta}\, q^\beta - q^\gamma D_\gamma q^\alpha G^{ij}_{\alpha\beta}\gamma^\beta - 3iq^\gamma D_\gamma q^\alpha \tilde{G}^{ij}_{\alpha\beta}\gamma^\beta\gamma_5\Big] + \\ +\mathcal{O}\Big(\frac{1}{q^5}\Big)\,, \qquad \text{(B.3)}$$

where (A is a color index in the adjoint representation)

$$G^{ij}_{\alpha\beta} = \frac{1}{2}\sum_A \lambda^A_{ij}G^{(A)}_{\alpha\beta}(0)\,, \qquad \tilde{G}^{ij}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\sigma}G^{\rho\sigma}_{ij}\,.$$

The terms of lowest dimension in the short-distance expansion of the quark propagator in eq. (B.2) which can contribute (*a priori*) to a tensor structure like the one in eq. (4.2) are those with two uncontracted powers of q-momenta in the numerator i.e., the last three terms in eq. (B.3). However, these terms, as far as their Dirac structure is concerned, are all proportional to one gamma matrix or to $\gamma^\beta\gamma_5$, (the last term). Therefore, when inserted in the r.h.s. of eq. (B.1), and taking into account that

$$Q\ Q_L^{(3)} = Q_L^{(3)}Q = \frac{1}{3}\,\mathrm{diag}(2,1,1)\,,$$

plus the identity

$$\gamma_\mu\gamma_\rho\gamma_\nu = g_{\mu\rho}\gamma_\nu + g_{\rho\nu}\gamma_\mu - g_{\mu\nu}\gamma_\rho - i\epsilon_{\mu\rho\nu\delta}\,\gamma^\delta\gamma_5\,, \qquad \text{(B.4)}$$

one finds that these terms cannot produce an antisymmetric tensor in the μ and ν indices. We then conclude that there is no contribution from *soft-gluons* with the tensor structure of eq. (4.2) with a power $p \le 3$.

B.2 Contribution from hard gluons

Expanding the T-product in eq. (4.1) in perturbation theory to $\mathcal{O}(g_s^2)$ generates the fol-

[11] See e.g. eq. (2.34) in ref. [44]. Notice that $i\tilde{S}^{ji}(q) = \int d^4z e^{-iq\cdot z} i\tilde{S}^{ji}(0,z)$.

lowing operator

$$
\begin{aligned}
\mathcal{U}_{\mu\nu}(q) = & \frac{i^2}{2!} g_s^2 \lim_{q\to\infty} \int d^4z\, e^{iq\cdot z} \int d^4x \int d^4y \, \times \\
& \times T\Big\{ : \bar{\psi}_i(x)\gamma^\alpha \frac{\lambda^A_{ij}}{2} G^{(A)}_\alpha(x)\psi_j(x) :: \bar{\psi}_k(y)\gamma^\beta \frac{\lambda^B_{kl}}{2} G^{(B)}_\beta(y)\psi_l(y) : \\
& : \bar{q}(z)\gamma_\mu Q q(z) :: \bar{q}(0)\gamma_\nu\gamma_5 Q_L^{(3)} q(0) : \Big\}\,.
\end{aligned}
$$

The contraction of the two gluon fields generates four possible classes of *hard* topologies illustrated in figure 3 above. Using the identity

$$
\sum_A \frac{\lambda^A_{ij}}{2}\frac{\lambda^A_{kl}}{2} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right),
$$

and the fact that the second term in this identity generates subleading terms in the $1/N_c$-expansion, the contributions from the four *hard* topologies can be written as follows:

$$
\begin{aligned}
\mathcal{U}^{(1)}_{\mu\nu}(q)\Big|_{q\to\infty} &= -i\frac{i^2}{2!}g_s^2\frac{1}{q^6}\, g^{\alpha\beta} : \left\{\bar{q}^i\,(S_{\mu\alpha}-A_{\mu\alpha})\,Q\,\psi^j\,\bar{q}^j\,(S_{\nu\beta}-A_{\nu\beta})\,T_3\,\gamma_5\psi^i\right\}(0): \\
\mathcal{U}^{(2)}_{\mu\nu}(q)\Big|_{q\to\infty} &= -i\frac{i^2}{2!}g_s^2\frac{1}{q^6}\, g^{\alpha\beta} : \left\{\bar{\psi}^i\,(S_{\mu\alpha}+A_{\mu\alpha})\,Q\,q^j\,\bar{\psi}^j\,(S_{\nu\beta}+A_{\nu\beta})\,T_3\,\gamma_5 q^i\right\}(0): \\
\mathcal{U}^{(3)}_{\mu\nu}(q)\Big|_{q\to\infty} &= +i\frac{i^2}{2!}g_s^2\frac{1}{q^6}\, g^{\alpha\beta} : \left\{\bar{q}^i\,(S_{\mu\alpha}-A_{\mu\alpha})\,Q\,\psi^j\,\bar{\psi}^j\,(S_{\nu\beta}+A_{\nu\beta})\,T_3\,\gamma_5 q^i\right\}(0): \\
\mathcal{U}^{(4)}_{\mu\nu}(q)\Big|_{q\to\infty} &= +i\frac{i^2}{2!}g_s^2\frac{1}{q^6}\, g^{\alpha\beta} : \left\{\bar{\psi}^i\,(S_{\mu\alpha}+A_{\mu\alpha})\,Q\,q^j\,\bar{q}^j\,(S_{\nu\beta}-A_{\nu\beta})\,T_3\,\gamma_5\psi^i\right\}(0):
\end{aligned}
$$

where $S_{\mu\alpha}$ and $A_{\mu\alpha}$ denote the tensors:

$$
S_{\mu\alpha} = q_\mu\gamma_\alpha + q_\alpha\gamma_\mu - g_{\mu\alpha}\,\not{q}\,, \qquad \text{and} \qquad A_{\mu\alpha} = iq^\eta\epsilon_{\mu\eta\alpha\sigma}\gamma^\sigma\gamma_5\,.
$$

Only the antisymmetric tensors contribute in the total sum, with the result

$$
\mathcal{U}_{\mu\nu}(q)|_{q\to\infty} = -i\frac{i^2}{2!}g_s^2\frac{1}{q^6}g^{\alpha\beta}4(i)^2q^\eta q^\lambda\,\epsilon_{\mu\eta\alpha\sigma}\,\epsilon_{\nu\lambda\beta\rho}(\bar{q}^i\gamma^\sigma\gamma_5 Q\psi^j)(\bar{q}^j\gamma^\rho T_3\psi^i)(0)\,.
$$

Fierzing the resulting four-quark operator, using the relevant terms of the identity

$$
(\bar{q}^i\gamma^\alpha\gamma_5 q^j)(\bar{\psi}^j\gamma^\beta\psi^i) = -\frac{1}{8}\epsilon^{\alpha\beta\rho\sigma}[(\bar{q}\psi)(\bar{\psi}\sigma_{\rho\sigma}q) + (\bar{q}\sigma_{\rho\sigma}\psi)(\bar{\psi}q)] + \cdots\,,
$$

and contracting two of the epsilon tensors (using the correct sign!) one finally gets the result

$$
\mathcal{U}_{\mu\nu}(q) = i\left[q_\delta\epsilon_{\mu\nu\alpha\beta}q^\alpha - q_\beta\epsilon_{\mu\nu\alpha\delta}q^\alpha\right]\left(-2\pi^2\frac{\alpha_s}{\pi}\right)\frac{\mathcal{O}^{\beta\delta}(0)}{(Q^2)^3} + \cdots\,;
$$

with $\mathcal{O}^{\beta\delta}(0)$ the tensor

$$
\mathcal{O}^{\beta\delta}(0) = \left[\frac{2}{3}\left(\bar{u}\sigma^{\beta\delta}u\right)(\bar{u}u) + \frac{1}{3}\left(\bar{d}\sigma^{\beta\delta}d\right)(\bar{d}d) + \frac{1}{3}\left(\bar{s}\sigma^{\beta\delta}s\right)(\bar{s}s)\right](0)\,.
$$

C. The OPE beyond the chiral limit

In the presence of quark masses, there are two types of terms that one has to consider in the evaluation of the function in eq. (4.1). (Recall that the relevant terms in the OPE are those with the explicit tensor structure shown in eq. (4.2).) There are terms of $\mathcal{O}(m)$ from the expansion of the quark propagator:

$$\frac{1}{\not{q}-m}=\frac{1}{\not{q}}+\frac{1}{\not{q}}m\frac{1}{\not{q}}+\mathcal{O}(m^2)\,,$$

and terms from the Taylor expansion of the quark field

$$q(x)=q(0)+x^\sigma D_\sigma q(0)+\cdots\,.$$

The first type expansion leads to a potential contribution:

$$\mathcal{U}_{\mu\nu}(q)|_m=i\epsilon_{\mu\nu\alpha\beta}\frac{1}{Q^2}\bar{q}(0)\sigma^{\alpha\beta}\mathcal{M}QQ_L^{(3)}q(0)\,, \tag{C.1}$$

where $\mathcal{M}$ denotes the quark mass matrix

$$\mathcal{M}=\mathrm{diag}(m_u,m_d,m_s)\,. \tag{C.2}$$

This contribution, however, does not have the two powers of q-momentum required by the tensor structure in eq. (4.2).

The Taylor expansion from the quark field leads to the operator

$$\mathcal{U}_{\mu\nu}(q)\big|_q=D_\rho\bar{q}(0)\gamma_\mu\frac{\gamma^\rho q^2-2\not{q}q^\rho}{Q^4}\gamma_\nu\gamma_5QQ_L^{(3)}q(0)-\bar{q}(0)\gamma_\nu\gamma_5\frac{\gamma^\rho q^2-2\not{q}q^\rho}{Q^4}\gamma_\mu Q_L^{(3)}Q\;D_\rho q(0)\,. \tag{C.3}$$

Using the Dirac equation

$$(iD^\rho\gamma_\rho-\mathcal{M})q=0\,, \tag{C.4}$$

the terms proportional to γ^ρ in eq. (C.3) lead to the operator

$$\mathcal{U}_{\mu\nu}(q)\big|_{\gamma^\rho}=-\frac{2}{Q^2}\left[D_\mu\bar{q}(0)\gamma_\nu\gamma_5QQ_L^{(3)}q(0)-\bar{q}(0)\gamma_\nu\gamma_5QQ_L^{(3)}D_\mu q(0)\right], \tag{C.5}$$

where we have used the fact that terms $\sim\sigma^{\alpha\beta}$ cancel with those from the quark-propagator contribution in eq. (C.1). The other terms in eq. (C.3) are proportional to $\not{q}$, and lead to the following combination of operators

$$\begin{aligned}\mathcal{U}_{\mu\nu}(q)\big|_{\not{q}}=-\frac{2}{Q^4}\Big\{&iq_\beta\epsilon_{\mu\nu\alpha\delta}q^\alpha\left(D^\beta\bar{q}(0)\gamma^\delta QQ_L^{(3)}q(0)+\bar{q}(0)\gamma^\delta QQ_L^{(3)}D^\beta q(0)\right)+\\&+q^\beta(q_\mu\delta^\beta_\delta-g_{\mu\nu}q^\delta+q_\nu\delta^\delta_\mu)\Big(D_\beta\bar{q}(0)\gamma_\delta\gamma_5QQ_L^{(3)}q(0)-\\&-\bar{q}(0)\gamma_\delta\gamma_5QQ_L^{(3)}D_\beta q(0)\Big)\Big\}.\end{aligned} \tag{C.6}$$

When inserting the operators $\mathcal{U}_{\mu\nu}(q)|_{\gamma^\rho}$ and $\mathcal{U}_{\mu\nu}(q)|_{\not{q}}$ in the T-product in the r.h.s. of eq. (2.1) there appear the following two-point functions

$$\Xi^{\beta\delta}{}_\rho(k)=\int d^4y e^{-ik\cdot y}\langle\Omega|T\left\{\left(D^\beta\bar{q}\gamma^\delta QQ_L^{(3)}q+\bar{q}\gamma^\delta QQ_L^{(3)}D^\beta q\right)(0)V_\rho^{\mathrm{em}}(y)\right\}|\Omega\rangle\,, \tag{C.7}$$

and

$$\Delta_{\beta\delta\rho}(k) = \int d^4y e^{-ik\cdot y}\langle\Omega|T\left\{\left(D_\beta\ \bar{q}\gamma_\delta\gamma_5 QQ_L^{(3)}q - \bar{q}\gamma_\delta\gamma_5 QQ_L^{(3)}D_\beta q\right)(0)V_\rho^{\mathrm{em}}(y)\right\}|\Omega\rangle\,. \tag{C.8}$$

Lorentz covariance and gauge invariance of the electromagnetic current restrict these two-point functions to the following decomposition[12]

$$\Xi_{\beta\delta\rho}(k) = -(g_{\delta\rho}k^2 - k_\delta k_\rho)k_\beta\Pi_{V'V}(k^2)\,, \tag{C.9}$$

and

$$\Delta_{\beta\delta\rho}(k) = i\epsilon_{\beta\rho\delta\alpha}k^\alpha\Pi_{A'V}(k^2)\,. \tag{C.10}$$

We can now obtain the asymptotic behaviour of the Green's function $W_{\mu\nu\rho}(q,k)$ defined in eq. (2.1) in terms of the invariant functions $\Pi_{V'V}(k^2)$ and $\Pi_{A'V}(k^2)$, with the result

$$\begin{aligned}\lim_{q\to\infty} W_{\mu\nu\rho}(q,k) &= -\frac{2i}{Q^2}\epsilon_{\mu\rho\nu\alpha}k^\alpha\Pi_{A'V}(k^2) - \\ &\quad -\frac{2i}{Q^4}\Big\{(q_\mu\epsilon_{\beta\rho\nu\alpha} + q_\nu\epsilon_{\beta\rho\mu\alpha})\,q^\beta k^\alpha\Pi_{A'V}(k^2) + \\ &\qquad + \epsilon_{\mu\alpha\nu\beta}\left(\delta_\rho^\beta k^2 - k^\delta k_\beta\right)q^\alpha(q\cdot k)\Pi_{V'V}(k^2)\Big\}\,. \end{aligned} \tag{C.11}$$

The next step consists in extracting from the expression in eq. (C.11) the transverse component: $\lim_{q\to\infty} W_{\mu\nu\rho}^{\mathrm{trans}}(q,k)$, defined in eq. (2.6). Taking the derivative with respect to the k-momentum, as defined in eq. (2.6), we finally get

$$\lim_{q\to\infty} W_{\mu\nu\rho\sigma}(q) = -\frac{2i}{Q^4}\left[q_\rho\epsilon_{\mu\nu\alpha\sigma}q^\alpha - q_\sigma\epsilon_{\mu\nu\alpha\rho}q^\alpha\right]\Pi_{A'V}(0)\,.$$

It follows then that the asymptotic behaviour of the function $W(Q^2)$, in the presence of quark masses, is given by

$$\lim_{Q^2\to\infty} W(Q^2) = -\frac{2}{Q^4}\Pi_{A'V}(0)\,.$$

What do we know about $\Pi_{A'V}(0)$? It turns out that the invariant functions $\Pi_{A'V}(k^2)$, $\Pi_{V'V}(k^2)$ defined in eqs. (C.9), (C.10) and the invariant function $\Pi_{VT}(k^2)$ defined in eq. (4.6) are not independent. This can be seen starting with the identity

$$i\bar{q}\sigma_{\alpha\beta}\mathcal{M}QQ_L^{(3)}q = \frac{1}{2}\left(D^\rho\bar{q}\gamma_\rho\sigma_{\alpha\beta}QQ_L^{(3)}q - \bar{q}\sigma_{\alpha\beta}\gamma_\rho QQ_L^{(3)}D^\rho q\right),$$

which follows from the Dirac equation (C.4), and using the Dirac algebra identity in eq. (B.4) which brings in the relevant operators. The resulting relation, to lowest order in the quark masses, is

$$2\,\mathrm{tr}\left(\mathcal{M}QQ_L^{(3)}Q\right)\Pi_{VT}(k^2) = \frac{1}{2}k^2\Pi_{V'V}(k^2) + \Pi_{A'V}(k^2)\,,$$

which at $k^2=0$ reduces to

$$\Pi_{A'V}(0) = \frac{2}{9}(4m_u - m_d - m_s)\Pi_{VT}(0)\,.$$

[12]There is in fact a second possible term in $\Xi_{\beta\delta\rho}(k)$ of the type $-(g_{\beta\rho}k^2 - k_\beta k_\rho)\ k_\delta\ \tilde{\Pi}_{V'V}(k^2)$, which however vanishes as can be seen by contracting the β and ρ indices in the r.h.s. of eq. (C.7), using translation invariance and conservation of V_ρ^{em}.

D. On the presence of $\log M_Z$ terms in $a_\mu^{EW(2)}(e,\mu,u,d,s,c)$

We have noticed in section 7.2 that for the two first generations, the $\log M_Z$ terms do not cancel in $a_\mu^{EW(2)}(e,\mu,u,d,s,c)$, see eq. (7.5). This result might appear as puzzling, since in the case of free quarks, which encompasses the constituent quark model, the cancellation of the $\log M_Z$ terms in $a_\mu^{EW(2)}(e,\mu,u,d,s,c)$ occurs, as observed in ref. [23], and as shown in eq. (7.7). However, this latter situation is conceptually very different from the one described by *confined* light (or even massless) quarks. This difference, that needs perhaps to be explained more in detail, is at the origin of the non-cancellation of the $\log M_Z$ contributions in eq. (7.5).

In order to proceed, let us start from eq. (5.7), that we write in a slightly different, but more convenient, form,

$$\begin{aligned} w_{\chi\mathrm{QM}}(z) &= \frac{N_c}{12\pi^2}\frac{8}{3}g_A\frac{M_\rho^2}{M_Q^2}\times\int_0^1 dy\int_0^{1-y}dx\,\frac{xy-y(1-y)}{1+\frac{Q^2}{M_Q^2}y(1-y)} \\ &= \int_{-\infty}^{\infty}dt\,\frac{1}{t+Q^2-i\varepsilon}\,\frac{1}{\pi}\,\mathrm{Im}\,w_{\chi\mathrm{QM}}(t)\,. \end{aligned} \tag{D.1}$$

The second, i.e. a dispersive form, is obtained upon performing the integration over the Feynman parameter x in the expression on the first line, followed by an obvious change of variable in the remaining integration over y, with the outcome

$$\frac{1}{\pi}\,\mathrm{Im}\,w_{\chi\mathrm{QM}}(t) = -\frac{1}{2}\,\frac{M_Q^2}{t^2}\,\frac{1}{\sqrt{1-4M_Q^2/t}}\,\theta(t-4M_Q^2)\left(\frac{N_c}{12\pi^2}\frac{8}{3}g_A M_\rho^2\right)\,. \tag{D.2}$$

For the case of a massless free fermion, one obtains

$$\lim_{M_Q\to 0} w_{\chi\mathrm{QM}}(z) = \frac{-N_c}{12\pi^2}\frac{2}{3}g_A\frac{M_\rho^2}{Q^2-i\varepsilon}\,, \tag{D.3}$$

since, as is well known,

$$\lim_{M_Q\to 0} -\frac{1}{2}\,\frac{M_Q^2}{t^2}\,\frac{1}{\sqrt{1-4M_Q^2/t}}\,\theta(t-4M_Q^2) = -\frac{1}{4}\,\delta(t)\,. \tag{D.4}$$

Thus, in the massless limit, the function $w_{\chi\mathrm{QM}}(z)$ reduces to a simple pole. This singularity at $Q^2 = 0$ also provides the $1/Q^2$ behaviour at large Q^2, which in turn will lead to the cancellation of the $\log M_Z$ terms. Notice also that this asymptotic $1/Q^2$ behaviour, and the ensuing cancellation of $\log M_Z$ terms, is preserved for a massive fermion, as shown by eq. (5.7) or by the dispersive representation above.

The real question we now have to address is how much of all this can be taken over to QCD. Let us first discuss the chiral limit. One immediate and important observation one has to make in the case of QCD is that the pole we have just discussed in the case of a massless free fermion no longer occurs. Indeed, one may evaluate the relevant three-point function $W_{\mu\nu\rho}(q_1,q_2)$ within the effective low-energy chiral lagrangian. The lowest order contribution starts at order $\mathcal{O}(p^4)$ and is given by the anomaly pole, as discussed in ref. [22]. At NLO, $\mathcal{O}(p^6)$ in this case, we have a counterterm contribution,[13] which leads to a non vanishing, but *constant*, value of $w(Q^2)$.[14] Thus, confinement drastically modifies the long distance behaviour of $W(Q^2)$ as compared to the free fermion case, not really a surprise. It now remains to discuss the short distance aspects. We may write the function $w(Q^2)$ in the dispersive form given before for the free fermion case,

$$w(Q^2) = \int_{-\infty}^{\infty} dt \, \frac{1}{t + Q^2 - i\varepsilon} \frac{1}{\pi} \operatorname{Im} w(t) \,, \tag{D.5}$$

but where now the perturbative QCD part describes the absorbtive part above a certain threshold s_0, which has to be sufficiently large so that perturbative QCD can be applied. Within the large-N_C framework, we may thus write

$$\frac{1}{\pi} \operatorname{Im} w(t) = \frac{1}{\pi} \operatorname{Im} w^{\mathrm{Res}}(t) + \frac{1}{\pi} \operatorname{Im} w^{pQCD}(t) \,. \tag{D.6}$$

The non perturbative piece from the narrow width resonant states writes

$$\frac{1}{\pi} \operatorname{Im} w^{\mathrm{Res}}(t) = \sum_R M_\rho^2 \alpha_R \delta(t - M_R^2) \,, \tag{D.7}$$

and M_R the mass of the R narrow state, $M_R^2 < s_0$. The perturbative contribution for a massive light quark with a current algebra mass m_q, to lowest order in perturbative QCD (free quarks, $g_A = 1$), is

$$\frac{1}{\pi} \operatorname{Im} w^{pQCD}(t) = -\frac{1}{2} \frac{m_q^2}{t^2} \frac{1}{\sqrt{1 - 4m_q^2/t}} \, \theta(t - s_0) \left(\frac{N_c}{12\pi^2} \frac{8}{3} g_A M_\rho^2 \right) . \tag{D.8}$$

This gives then

$$w(z) = w^{\mathrm{Res}}(z) + w^{pQCD}(z) \,, \tag{D.9}$$

with

$$\begin{aligned}
w^{\mathrm{Res}}(z) &= \sum_R \frac{\alpha_R}{z + z_R} \,, \qquad z_R = \frac{M_R^2}{M_\rho^2} \,, \\
w^{pQCD}(z) &= -\frac{1}{4} \frac{M_\rho^2}{Q^2} \left[1 - \sqrt{1 - \frac{4m_q^2}{s_0}} - \frac{2m_q^2}{Q^2} \log \left(1 + \frac{Q^2}{s_0} \right) + \mathcal{O}(m_q^4) \right] \times \\
&\quad \times \left(\frac{N_c}{12\pi^2} \frac{8}{3} g_A \right) (1 + \mathcal{O}(\alpha_s)) \,.
\end{aligned}$$

We see here the crucial difference with the constituent quark model, where s_0 would coincide with $4M_Q^2$, thus providing the same leading $1/Q^2$ short distance behaviour as for a massless

[13] We stay of within the large-N_C framework, hence meson loops can be ignored.

[14] In terms of the $\mathcal{O}(p^6)$ chiral lagrangian constructed for the odd intrinsic parity sector in [45], the relevant low-energy constant is C_{22}^W.

free quark. In QCD, the onset of the continuum occurs at a scale s_0 that is much larger than the scale set by the light quark masses, and for $s_0 \gg 4m_q^2$, one finds

$$\lim_{z\to\infty} zw^{pQCD}(z) = \mathcal{O}(m_q^2)\,.$$

Thus, in the chiral limit, or if one considers only first order explicit chiral symmetry breaking effects, the short distance behaviour of $w(z)$ in QCD reads as given in eq. (6.10),

$$\lim_{z\to\infty} w(z) = -\frac{4}{9}\frac{(4m_u - m_d - m_s)\langle\bar\psi\psi\rangle_0}{Q^4} + \frac{16}{9}\pi^2\frac{\alpha_s}{\pi}\frac{\langle\bar\psi\psi\rangle_0^2}{Q^6} + \cdots\,.$$

The fact that the leading short distance fall off of $w(z)$ in QCD is $1/z^3$ in the chiral limit, or $1/z^2$ for massive light quark if only first order order explicit chiral symmetry breaking effects are retained, explains why the cancellation of $\log M_Z$ terms no longer occurs in QCD.

We conclude therefore that the cancellation of the $\log M_Z$ terms does not occur in QCD in the chiral limit, and by stressing once again very strongly that constituent quark models do not, in general, provide an adequate description of fundamental properties of QCD.

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